LECTURE 3 Principal Hypotheses and Assumptions in Mechanics of Materials. Deformations, Internal Forces and Stresses.

Fundamental principles of strength of materials are based on laws and theorems of general mechanics and generally on laws of solid bodies statics.

Theoretical mechanics deals with bodies which are called **absolute solid (rigid) bodies**. In contrast to theoretical mechanics, the mechanics of materials deals with **deformable solids**.

1 Principal Hypotheses in Mechanics of Materials

The principal hypotheses for deformable solids are following:

1. The material of a structure is considered to be **continuous** at all points of the body, i.e. we have got a continuous medium. Due to continuity's property it is possible to apply infinitesimal analysis to such a medium.

2. The material of structure is considered to be **homogeneous** at all points of the body, i.e. a body possesses the same properties at all points.

3. A continuous medium is taken to be **isotropic**, i.e. possessing the same properties in all directions. Isotropic materials include **metals**, **concrete**, and some types of **plastics**. Materials possessing different properties in different directions are called **anisotropic**. For example, **wood**, **composites** etc.

4. All bodies are assumed to be **ideally elastic** if their deformations return to zero after removing the applied forces (see Figs 1, 2).

5. **Deformations** of elastic bodies under the action of external loads are not significant comparing to the dimensions of bodies, i.e. at elastic deformation the body dimensions are not changed substantially. It allows applying the equilibrium equations without changing the geometrical dimensions of the structure, i.e. to apply external forces to non-deformable structure.

6. Generally, the displacements of solid bodies are proportional to acting forces within certain limits, i.e. the solid is assumed to be **linearly elastic** (Fig. 2). In nonproportional relationship between forces and displacements solid is assumed to be nonlinearly elastic (Fig. 1).

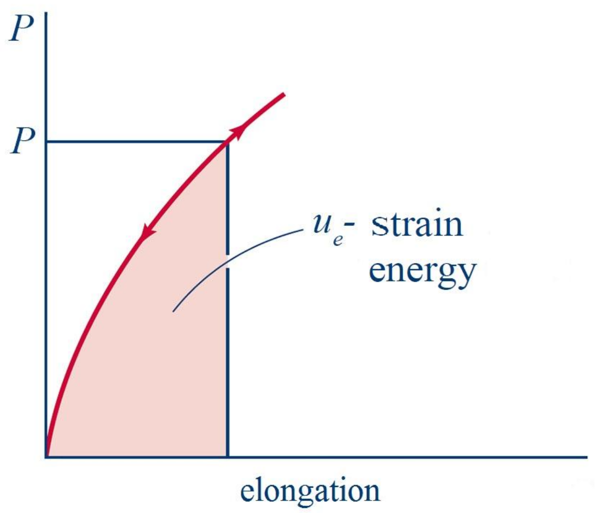

Fig. 1

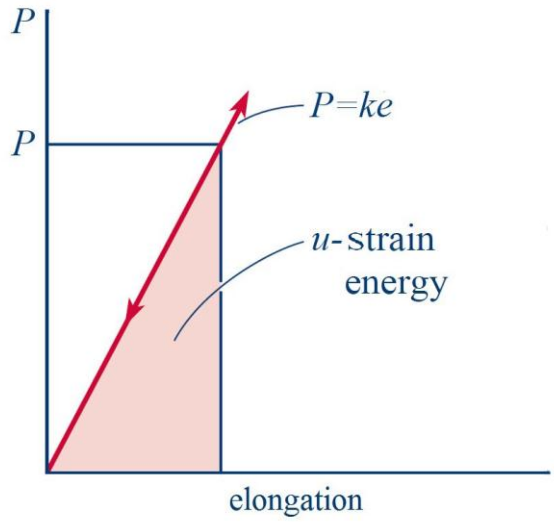

Fig. 2

2 Deformations

None of materials existing in nature is absolutely rigid. All natural and artificial materials change their dimensions and shapes due to external forces, i.e. they are **deformable**. There are two kinds of deformations:

a) **elastic deformation**, which disappears completely after load removing (Fig. 3);

b) **plastic deformation**, which doesn't disappear after load removing (Fig. 4).

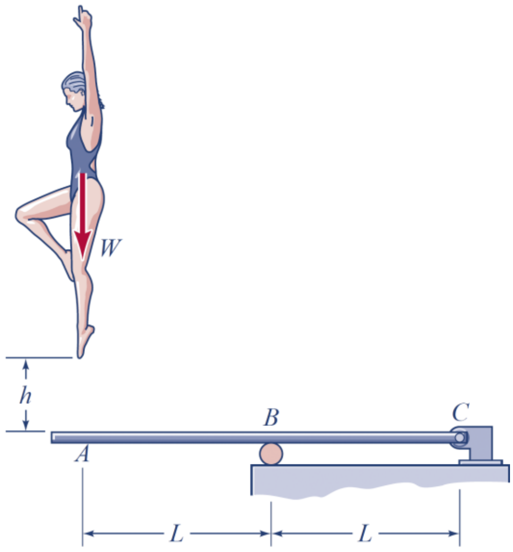

Fig. 3

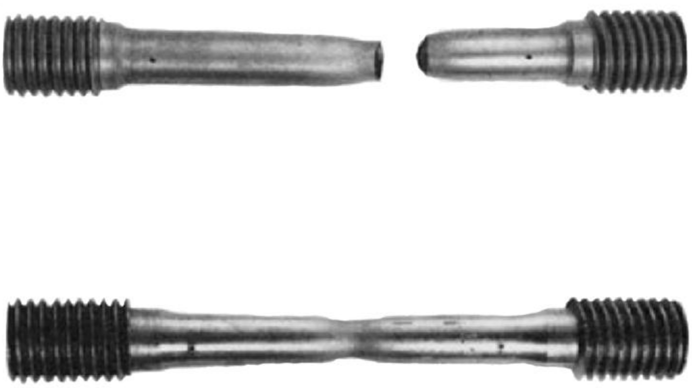

Fig. 4

3 External Forces

If a structure is considered separately from surrounding bodies it is affected by forces which are defined as **external forces**. According to the natural distribution of external forces over the body, the latter are divided into:

a) **body forces**, which are distributed over the volume of a body and are applied to each of its particles (body forces comprise, for example, the force of gravity, forces of magnetic attraction, centrifugal forces) (Fig. 5);

b) **surface forces**, which act on portions of surface (forces of wind, pressure) (Figs 6, 7, 8);

c) **forces distributing along a line** (Figs 9, 10);

d) **concentrated forces**, which are applied over the relatively small surface (Figs 11, 12);

e) **couples of forces** or concentrated moments (Figs 13, 14);

f) **distributed moments** (Figs 15, 16).

It is usual to distinguish **static loads**, **dynamic loads**, **cyclic loads**.

Static loads act on a structure continuously (see Fig. 17); dynamic loads act on a structure during short interval of time (see Figs 18, 19). Cyclic loads can be changed by a specified law (see Fig. 20).

4 Internal Forces

External forces produce strains and internal elastic forces which appear in all points of deformable solid and withstand changing the body shape and volume. Internal forces are proportional to external ones (**hypothesis of linear elasticity**).
If a solid body is in natural state (without external forces application), then internal forces are assumed to be zero (**hypothesis of non-stressed state**).

5 Method of Sections

The ***method of sections and equations of equilibrium enable to determine the magnitude of internal forces*** that appear under the action of external forces. Consider in Fig. 21 the application of method of sections to a body, balanced by a system of external forces

$$\vec{F}_1,\ \vec{F}_2, \ldots,\ \vec{F}_n. \tag{1}$$

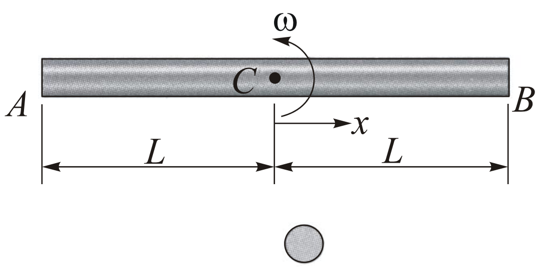

Fig. 5

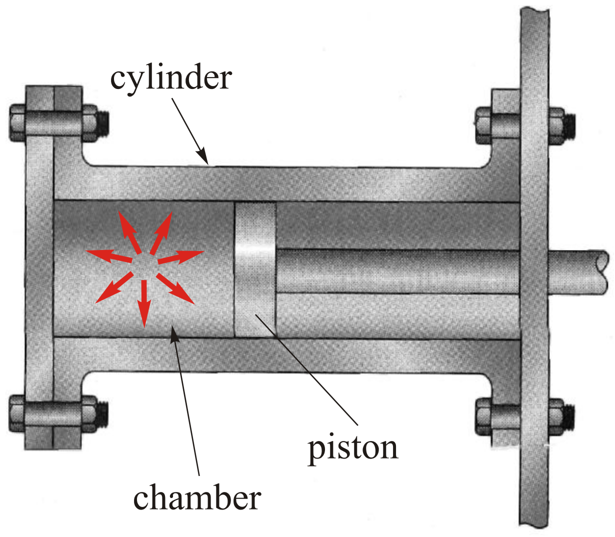

Fig. 6

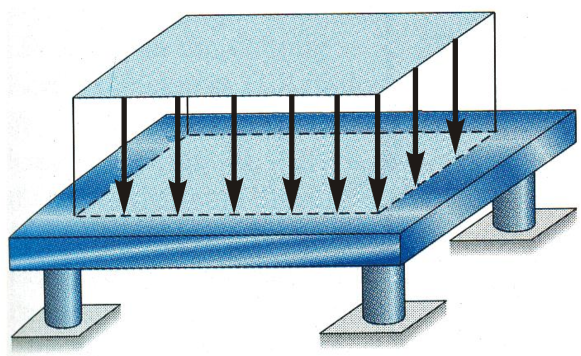

Fig. 7

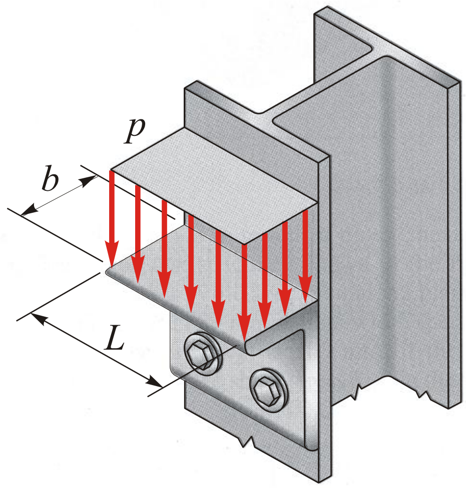

Fig. 8

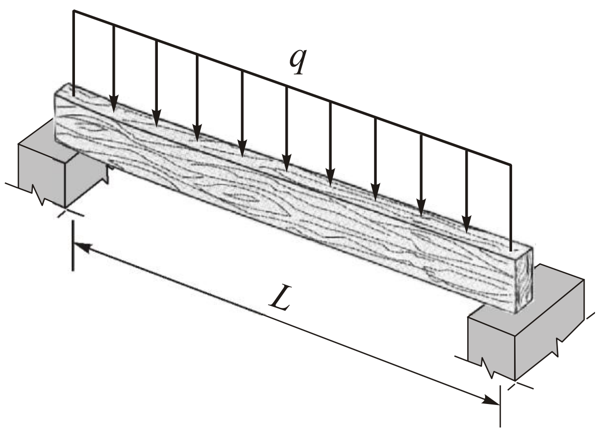

Fig. 9

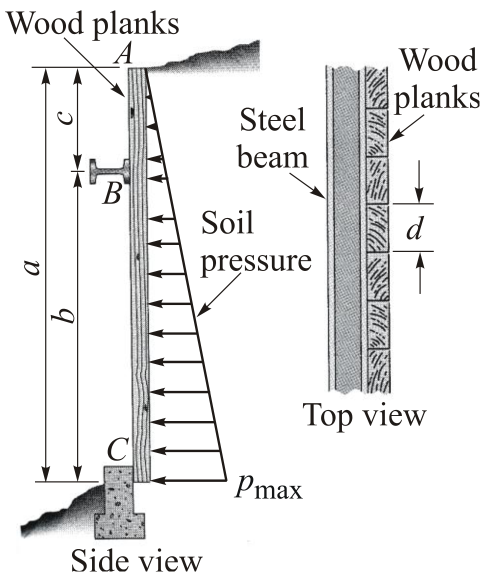

Fig. 10

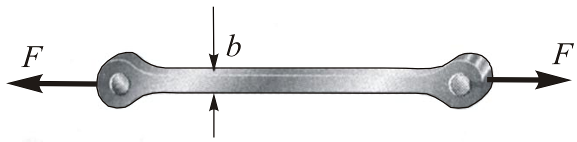

Fig. 11

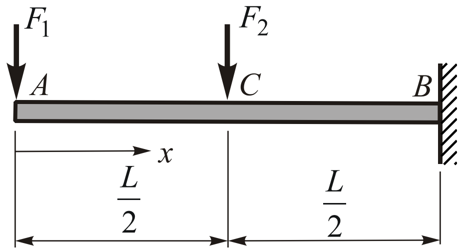

Fig. 12

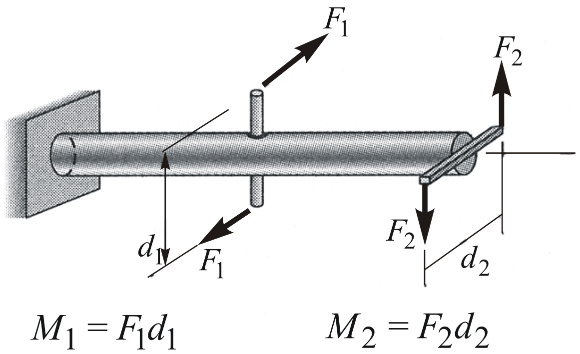

Fig. 13

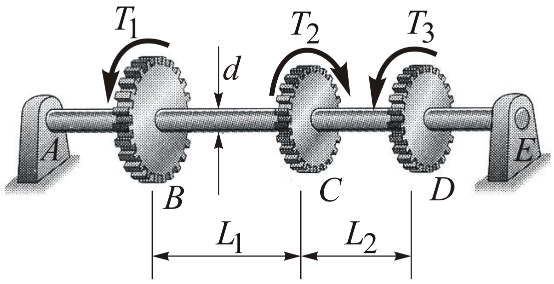

Fig. 14

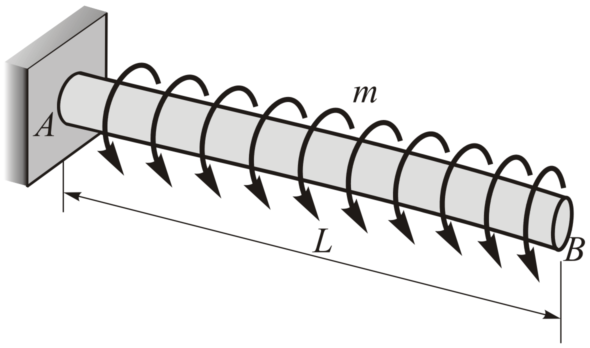

Fig. 15

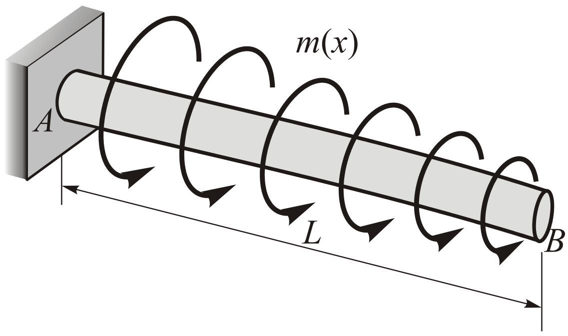

Fig. 16

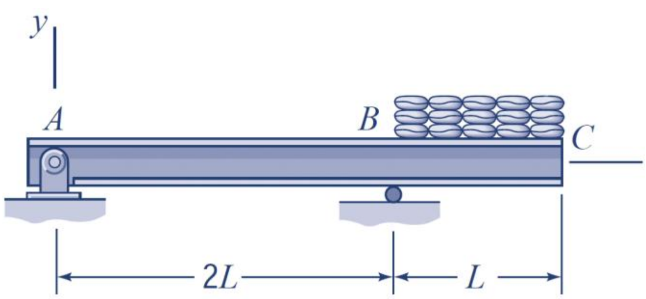

Fig. 17

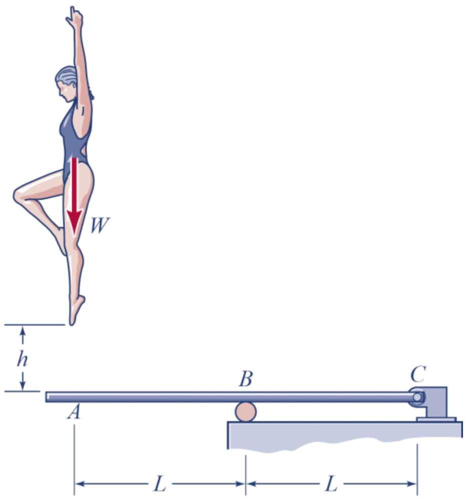

Fig. 18

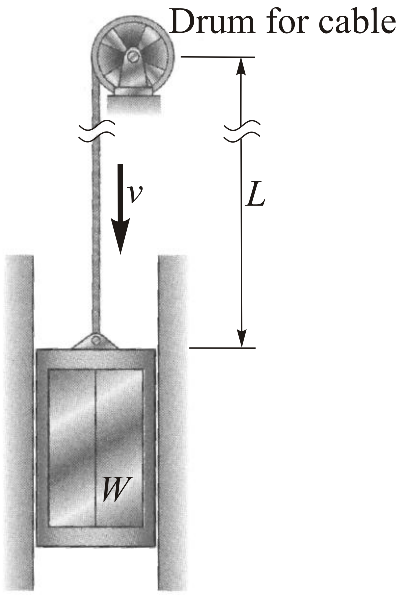

Fig. 19

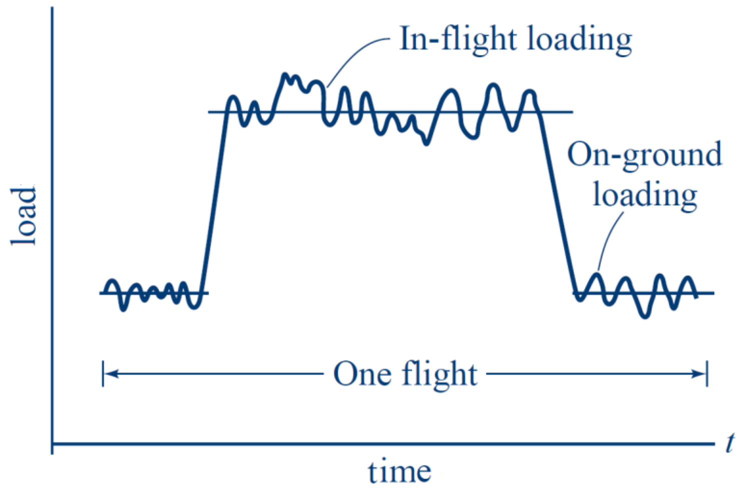

Fig. 20

It is evident, that the $\vec{F}_1$, $\vec{F}_2$, …, $\vec{F}_n$ are satisfying the conditions of equilibrium. The essence of method consists of the following four procedures:

1. The body analyzed is divided **mentally** by a plane including the point where the internal forces must be determined;

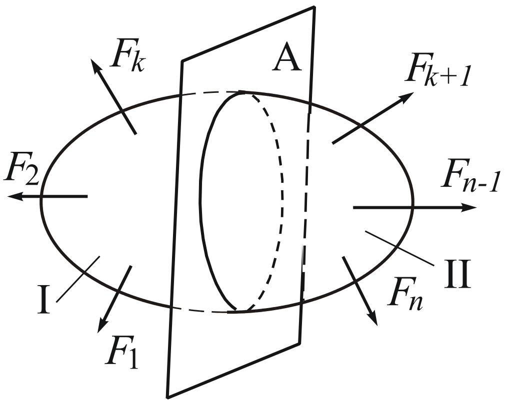

Fig. 21

2. One of the portions of the body is named as rejected part (part II in Fig. 21);

3. The effect of rejected part is replaced by internal forces so that the remaining part (I) is in equilibrium;

4. The equations of equilibrium are developed for the forces acting on the remaining part of the body to calculate internal forces in the cross-section.

Cut the body with a plane and reject the right-hand part, replacing the effect of the rejected part to the remaining left-hand part by means of internal elastic forces. $\vec{R}$, $\vec{M}_R$ are the **main vector** and **main moment** of internal forces by means of which the rejected part of the body acts on the remaining part according to third Newton's law (Fig. 22).

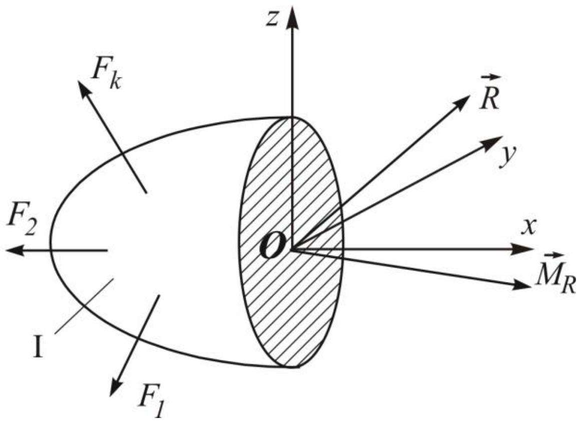

Fig. 22

It is usual to reduce $\vec{R}$ and $\vec{M}_R$ toward centroid of a section. We choose a system of coordinates *x*, *y*, *z*. Let *x* axis be normal toward the section with *y* and *z* axes within its plane (see Fig. 22).

The remaining part is in equilibrium, i.e. the system of forces

$$(\vec{F}_1,\ \vec{F}_2,\ \vec{F}_k,\ \ldots,\ \vec{R},\ \vec{M}_R) \qquad (2)$$

satisfies the conditions of equilibrium. By projecting the resultant force vector $\vec{R}$ and the resultant moment vector $\vec{M}_R$ on the axes *x*, *y* and *z* we obtain six components (three forces and three moments). These components of the main vector and main moment of internal forces, appearing on a cross section of a bar, are called **internal force factors** (**internal forces**) acting in that section.

The equilibrium conditions of the remaining part give us six equations of equilibrium, determining internal force factors:

$$\sum_{i=1}^{k}\vec{F}_i+\vec{R}=0; \qquad \sum_{i=1}^{k}\vec{M}_0\left(\vec{F}_i\right)+\vec{M}_R=0. \qquad (3)$$

or in projections:

$$N_x=\sum_{i=1}^{k}F_{ix}, \qquad M_x=\sum_{i=1}^{k}M_0\left(F_{ix}\right),$$

$$Q_y=\sum_{i=1}^{k}F_{iy}, \qquad M_y=\sum_{i=1}^{k}M_0\left(F_{iy}\right), \qquad (4)$$

$$Q_z=\sum_{i=1}^{k}F_{iz}, \qquad M_z=\sum_{i=1}^{k}M_0\left(F_{iz}\right),$$

where normal to the section component of internal forces N_x is called **normal force** in the section. The forces Q_y and Q_z lying in the cross-section are called **transverse or**

shearing forces. The moment directed toward normal axis, M_x, is called **twisting moment** (**torsional moment**). Moments M_y and M_z are called **bending moments** with respect to *y* and *z* axes (see Fig. 23).

When external forces are known, all internal force factors are obtained using six equilibrium equations – they can be composed for any cut portion of a rod (I or II).

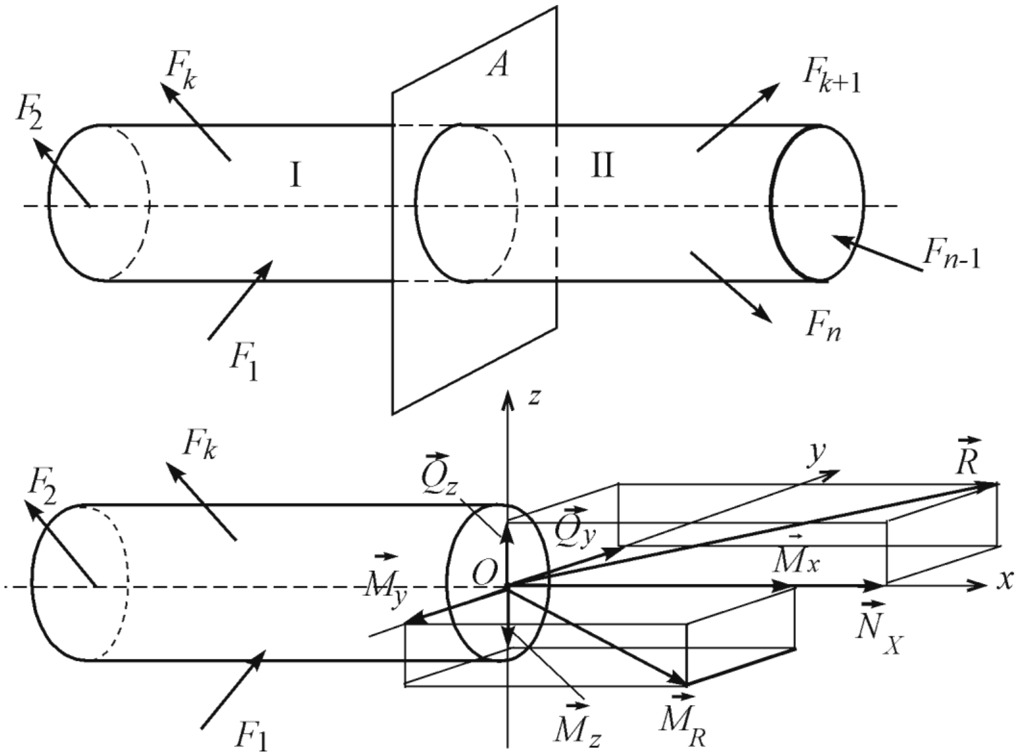

Fig. 23

In mechanics of materials we deal with the structural elements named as **rod**. Any structural element having one cross-sectional dimension (length) 10 times and more greater than remaining two ones will be assumed as a rod. An examples of engineering structures consisting of the rods are represented in Fig. 24.

Structural elements named as **plate** and **shell** are also considered in mechanics of materials (Figs 25, 26), but only under simplest types of loading.


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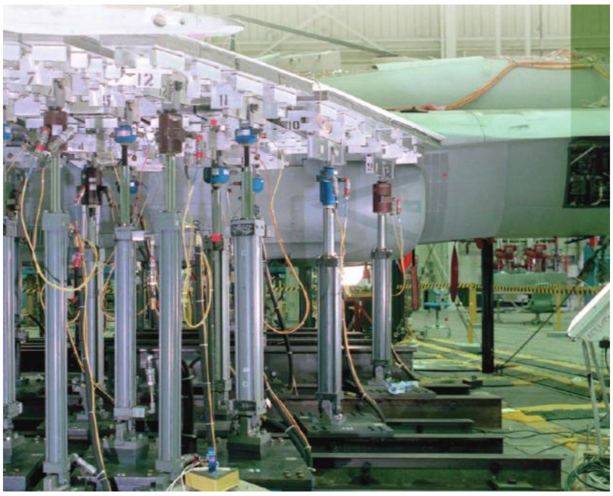

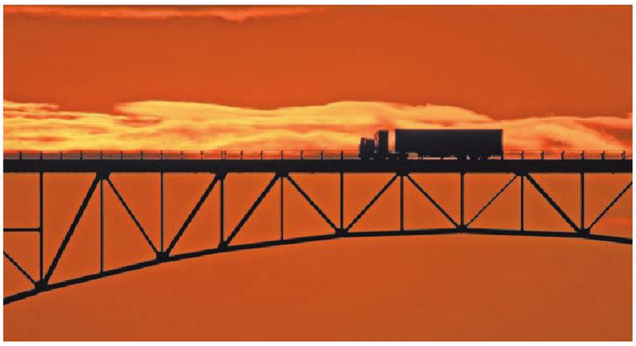

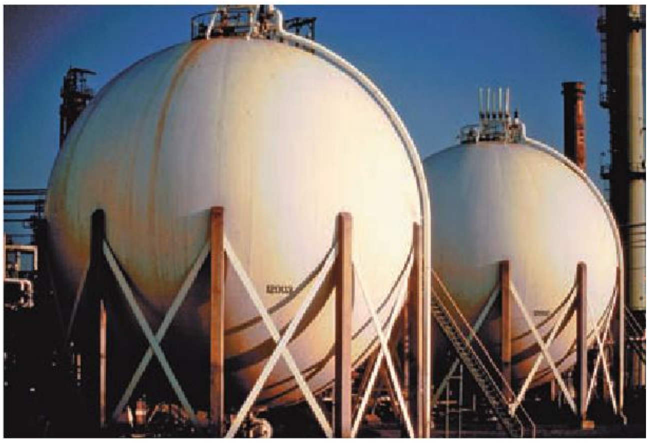

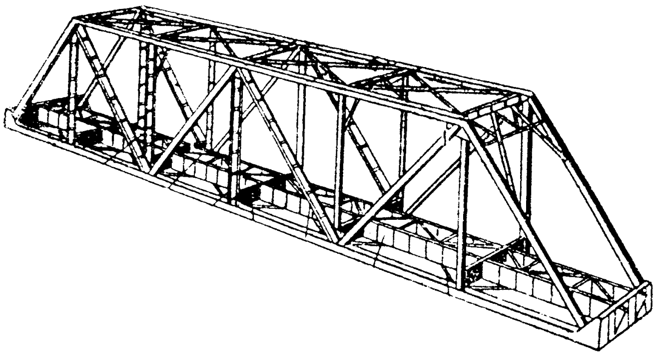

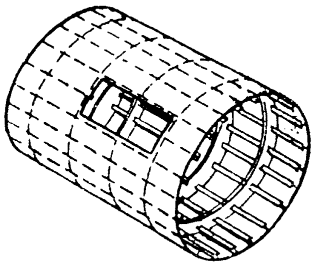

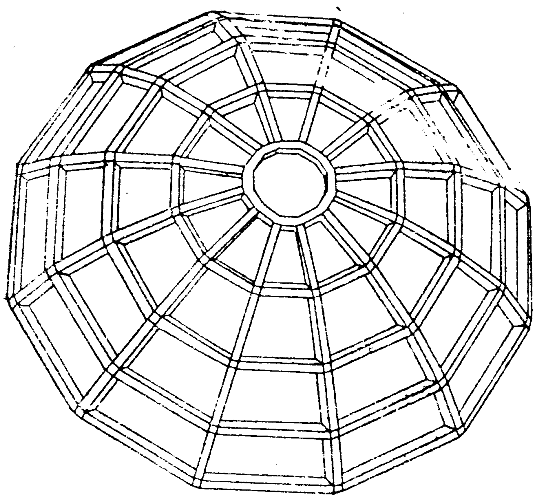

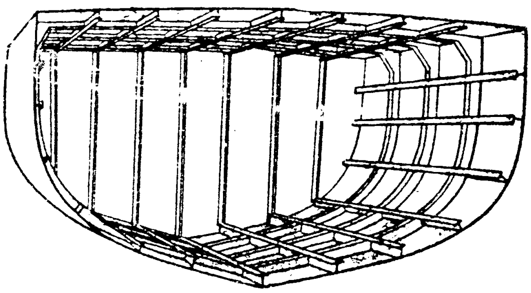

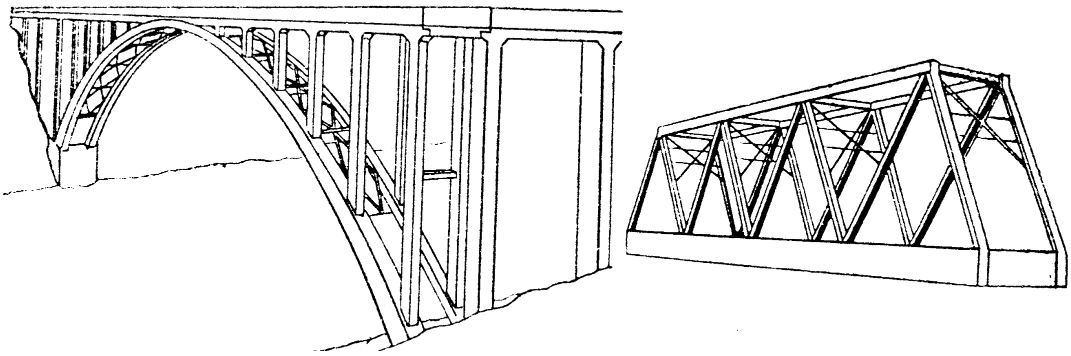

Fig. 24 Examples of structural elements named by a **rod**

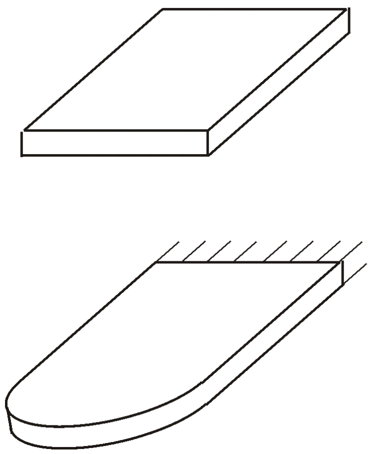

Fig. 25 Examples of structural elements named by a **plate**

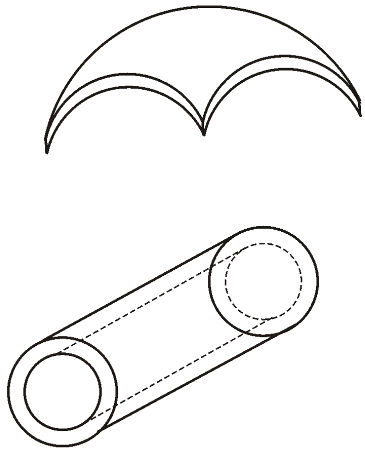

Fig. 26 Examples of structural elements named by a **shell**

6 Concept of Stress

In order to characterize the law of internal forces distribution over the section a measure of their intensity should be introduced. The measure used is **stress**.

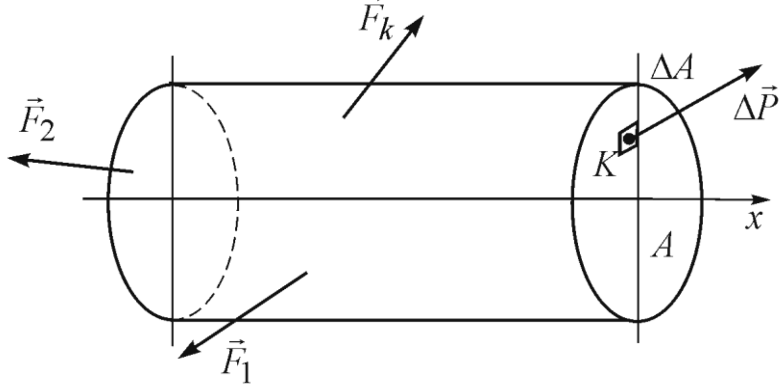

Fig. 27

Consider an arbitrary cross-section A of elastic body (Fig. 27). We isolate an element of area ΔA enclosing a point the internal force $\Delta P[\mathrm{N}]$ is acting on. The **average stress** on the area $\Delta A[\mathrm{m}^2]$ is taken to be

$$\vec{p}_{av} = \frac{\Delta\vec{P}}{\Delta A}. \tag{5}$$

We reduce the area ΔA by contracting it to the point K. Since the medium is continuous, a limiting process is applied as $\Delta A \to 0$. In the limit we obtain

$$\vec{p} = \lim_{\Delta A \to 0}\left(\frac{\Delta\vec{P}}{\Delta A}\right) = \frac{d\vec{P}}{dA}. \tag{6}$$

The vector quantity of **true stress** $\vec{p}$ represents the intensity of internal forces distribution at the point K within section A.

Stress has the dimension of force divided by area. The SI unit of stress is **Pascal**: $\mathrm{Pa} = \dfrac{\mathrm{N}}{\mathrm{m}^2}$ or **megaPascal**: $\mathrm{MPa} = 10^6\,\mathrm{Pa}$.

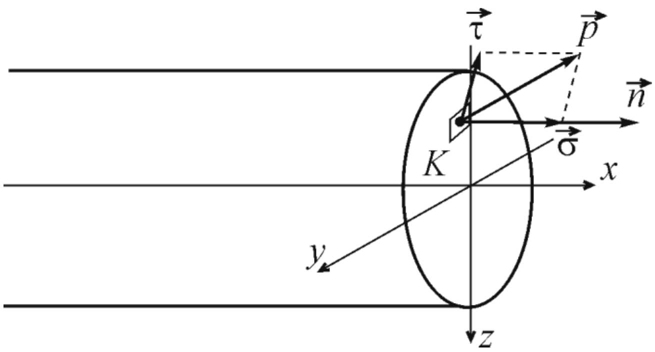

Fig. 28

The stress in given point of a section considered as a **vector quantity**, i.e. it is characterized by magnitude and direction.

Let us resolve the total stress vector into two components: one, directed along the normal to the section and the other one, lying in the section plane (see Fig. 28).

Stress component directed along the normal to the section is called **normal stress** and designated with greek letter σ. ***The component lying in the section plane will be called the*** **tangential (shearing) stress** and designated with greek letter τ.

7 Relations Between Stresses and Internal Forces

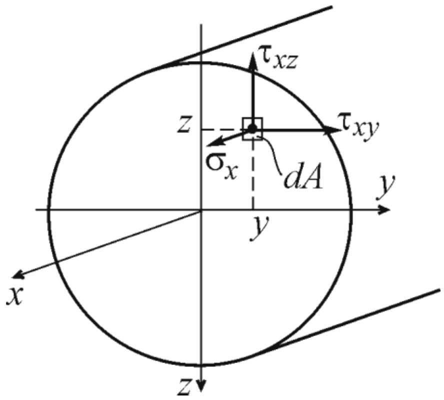

Fig. 29

In majority of cases, it is convenient to resolve the total stress into three components directed parallel to coordinate axes. For the point in the cross section of a bar it is shown in Fig. 29. For these components, the following rule of subscripts has been adopted: the ***first subscript corresponds the coordinate axis perpendicular to the plane***; the ***second subscript gives the coordinate axis to which the given stress is parallel***. According to this rule, normal stresses should be written with two subscripts, i.e. σ_{xx}, but usually one of two subscripts is omitted.

Let us establish how the stresses and internal force factors in a bar cross section are interrelated. Multiplying the stresses σ_x, τ_{xz} and τ_{xy} by the area *dA*, we obtain **elementary internal forces**:

$$\begin{aligned} dN_x &= \sigma_x dA, \\ dQ_y &= \tau_{xy} dA, \\ dQ_z &= \tau_{xz} dA. \end{aligned} \tag{7}$$

Summing these elementary forces over the area of the section we find expressions for **main vector components** of internal forces:

$$\begin{aligned} N_x &= \int_A \sigma_x dA, \\ Q_y &= \int_A \tau_{xy} dA, \\ Q_z &= \int_A \tau_{xz} dA. \end{aligned} \tag{8}$$

Multiplying each of the elementary forces by the distance to the corresponding axis, we obtain the **elementary moments** of internal forces:

$$dM_x = -\left(\tau_{xz} dA\right) y + \left(\tau_{xy} dA\right) z,$$
$$dM_y = \left(\sigma_x dA\right) z, \tag{9}$$
$$dM_z = \left(\sigma_x dA\right) y.$$

Summing the elementary moments over the entire area of the section, we get the expressions for **main moment components** of internal forces:

$$M_x = \int_A \left(\tau_{xy} z - \tau_{xz} y\right) dA,$$
$$M_y = \int_A \sigma_x z dA, \tag{10}$$
$$M_z = \int_A \sigma_x y dA.$$

These formulas will be used in the further discussion for solving one of the principal problems in the theory of strength of materials: ***the determination of stresses with known internal force factors***.

8 Saint-Venant's Principle

The features of external forces application are manifested, as a rule, at distances not exceeding the characteristic dimensions of bar cross section (see Fig. 30).

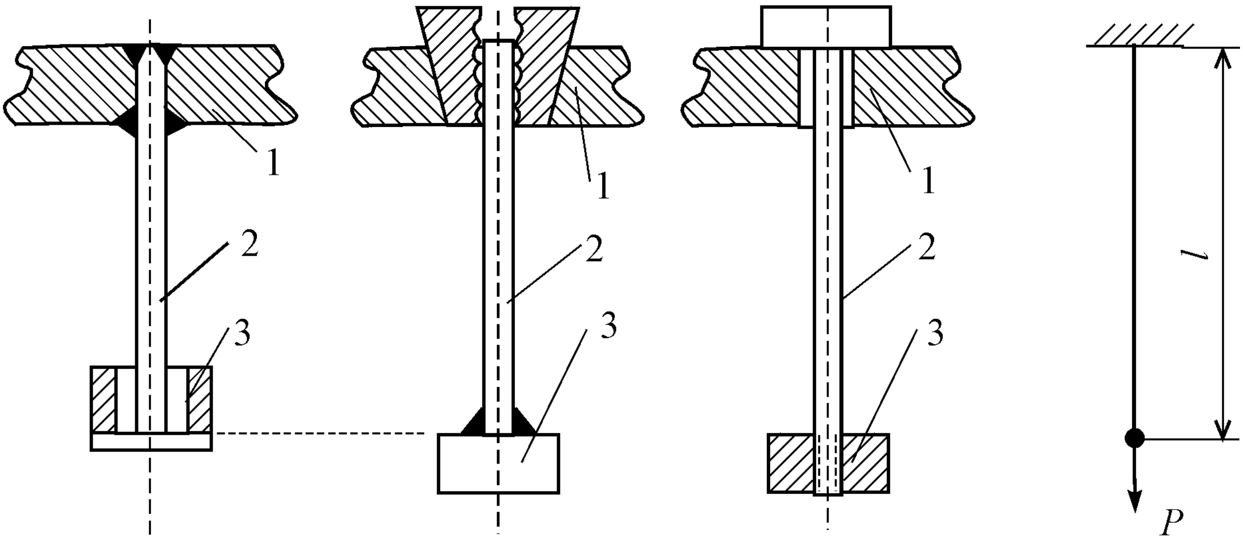

Fig. 30

Therefore one should neglect the portion of the bar located in the zone of external forces application if you will estimate internal stresses in the bar using the methods of mechanics of materials. Example of Saint-Venant’s principle is illustrated by Fig. 31, where vicinity of *P*-force application is the Saint-Venant's zone with non-uniform stress distribution.

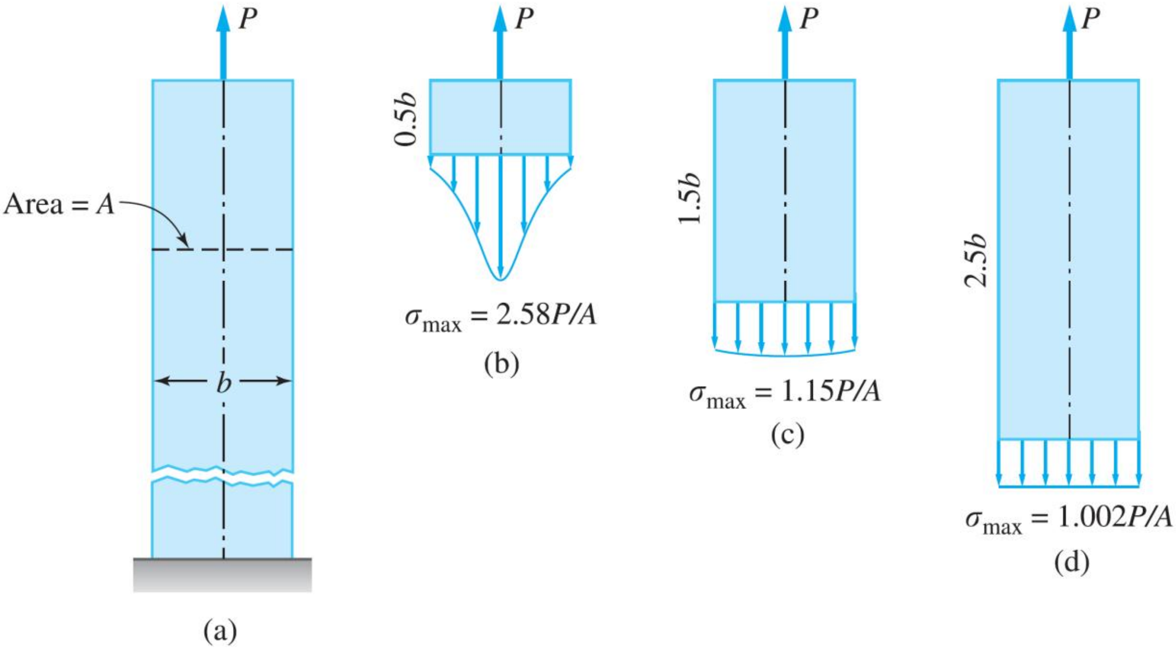

Fig. 31

9 Principle of Superposition

Since deformations the mechanics of materials deals with, are relatively small (elastic), ***external forces may be assumed to act independently from one another***, i.e. the deformations and internal forces appearing in elastic bodies do not depend on the order the external forces are applied in. Besides, the total effect of the whole system of forces acting on a body is assumed to be the sum of effects produced by each force separately. This is the **principle of superposition.**

10 Construction of Internal Force Factors Diagrams

A graph (plot) of internal force factor distribution along the length of bar is called diagram. There are some rules of significant importance for diagrams constructing:

1. Diagrams of internal force factors are always constructed on axial line which is parallel to axial line of a bar.
2. The force factor magnitude is laid off along a normal to the axis.
3. The diagrams include **denominations, dimensions, signs, numerical information**.