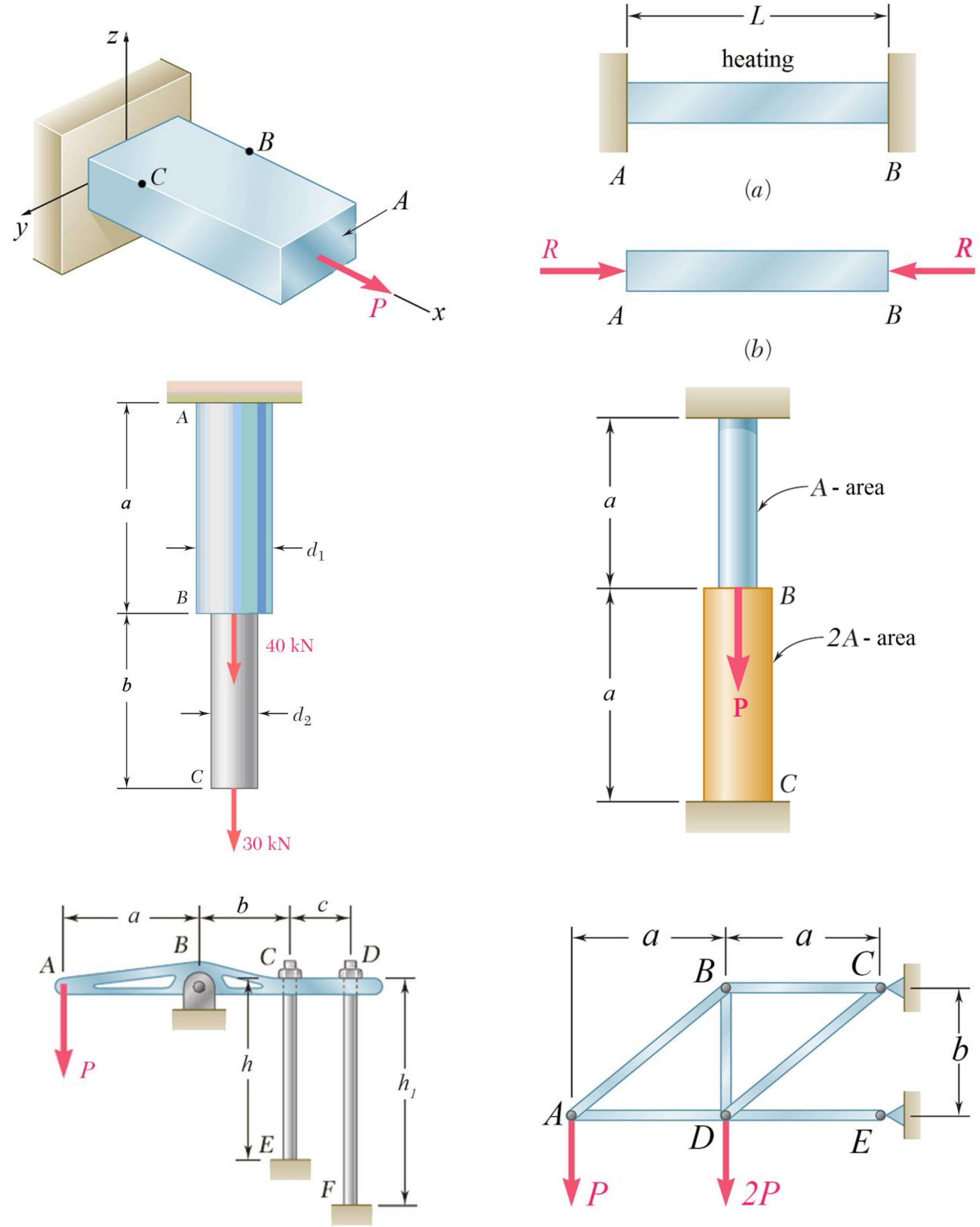
LECTURE 4 Tension-Compression and Torsional Deformations. Diagrams of Internal Forces

1 Tension-Compression Deformation. Diagrams of Normal Forces

In case ***only normal force occurs at cross sections*** of prismatic bar we have **tension** or **compression deformation**. Bar in tension-compression will be named as a **rod**. Examples of the rods and rod systems are shown in Fig. 1.



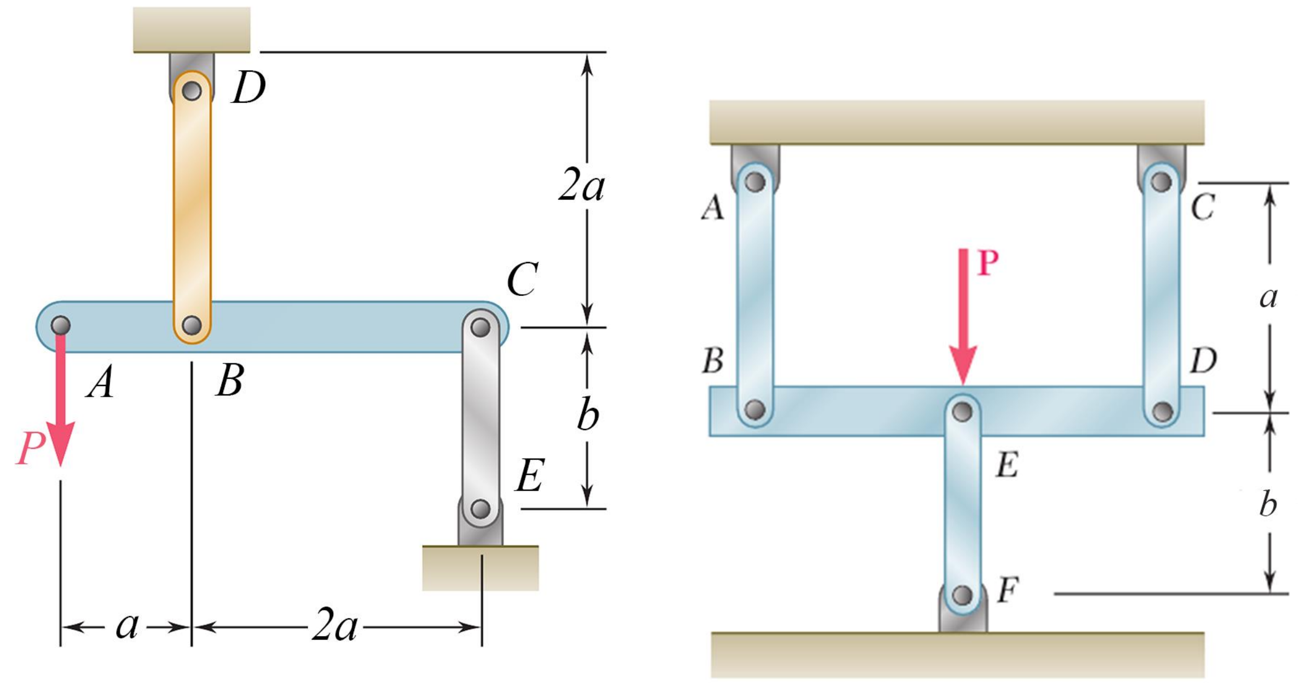


Fig. 1

Let us consider a bar of constant cross-sectional area (prismatic bar) with two oppositely directed equal forces applied to its ends:

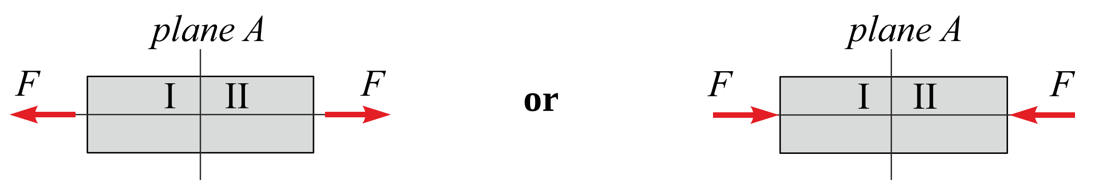


Fig. 2

Both in tension and compression, the bar is in equilibrium, i.e. the sum of all forces projections onto x axis is equal to zero. The normal force N_x can be found by the **method of sections** taking the concept of **free body diagram**: let the bar be virtually cuted in the point where the force N_x is to be determined and the effect of rejected portion of a bar onto the remaining portion to be replaced by internal force N_x.

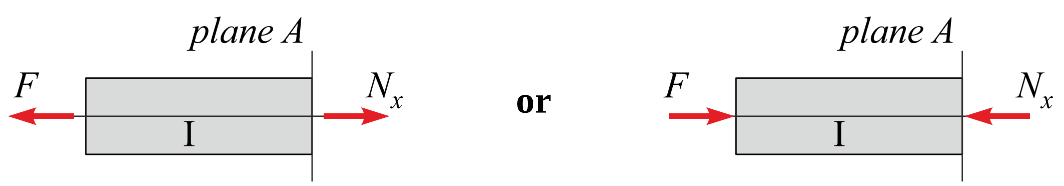


Fig. 3

In tension, the force N_x is directed away from the section, having plus sign. In compression, it is directed to the section, having minus sign.

Rule ***If the force N_x is directed away from the section then it has plus sign; if the force N_x is directed to the section, then it has minus sign.***

In the case when a few external forces are applied, the value of normal force in an arbitrary cross-section numerically equals to the algebraic sum of external forces applied to the part of a rod under consideration.

Example 1 Calculation of normal force in a rod in tension

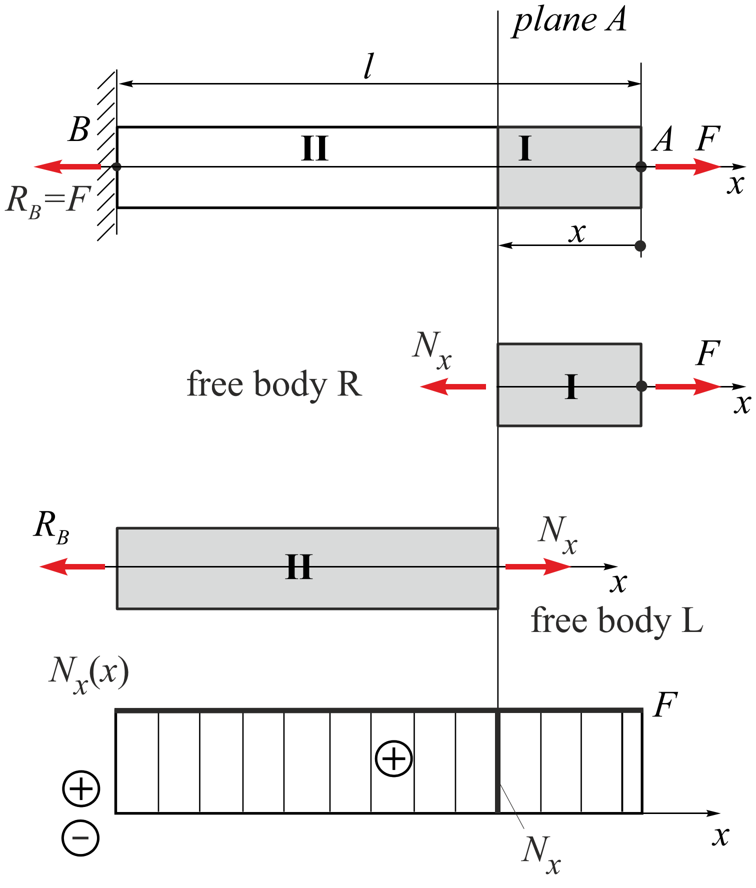


Fig. 4

Given: The bar of the total length l under the action of the force F.

R.D: the normal force along the length of the bar.

Consider the equilibrium of right remaining portion analyzing the free body diagram:

$$\sum F_x = 0, \quad F - N_x = 0,$$

$$N_x = +F.$$

In our case, $N_x = F = const$ and graph is a straight line with abrupt changes in the points of forces F, R_B application, i.e. at the ends of the bar, by the magnitude of forces $F = R_B$.

The same result corresponds to the equilibrium of left hand portion of tensile rod:

$$\sum F_x = 0, \quad N_x - R_B = 0, \qquad N_x = +R_B = +F.$$

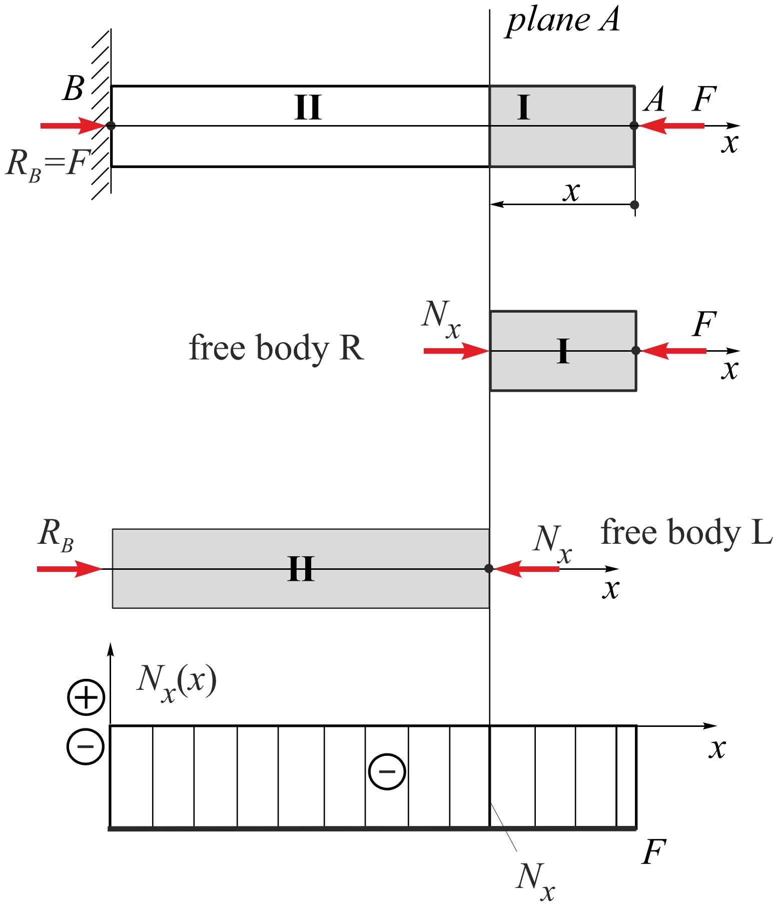


Fig. 5

Example 2 Calculation of normal force in compressed rod

In this case, $N_x = -F = const$ due to compressive (negative) deformation. The graph is a straight line, but $N_x < 0$.

For left hand portion its equation of equilibrium is

$$\sum F_x = 0, \quad -N_x + R_B = 0,$$

$$N_x = +R_B = +F.$$

Note. Equation of equilibrium allows normal force finding as its modulus. Its sign corresponds to physical sense of deformation: in tension $N_x > 0$, in compression $N_x < 0$.

Example 3 Calculation of normal forces in stepped prismatic bar

Given: Bar of a stepped profile: $F_1 = 60$ kN, $F_2 = 40$ kN, $F_3 = 70$ kN. **R.D.**: $N_x(x)$

The construction of normal forces diagram is to be started by division of bar into four portions I, II, III and IV according to the forces applied (see Fig. 6). Herewith, we begin the normal forces diagram construction from **free end** of the bar.

If a number of forces are applied to a bar, the internal force factor, i.e. the normal force N_x, can be found with the method of sections, using the conditions of equilibrium. In the section under consideration, the force N_x is determined as the algebraic sum of all forces acting on a bar up to the considered section:

$$N_x = \sum_{i=1}^{k} F_i,$$

where F_i is an i-th force acting on the bar.

If it turns out that $N_x > 0$, it will be directed away from section and be a tensile force; otherwise, it is directed to the section and is a compressive force:

$$N_x^I(x) = F_1 = +60\,\text{kN}, \qquad N_x^{II}(x) = F_1 = -60\,\text{kN},$$

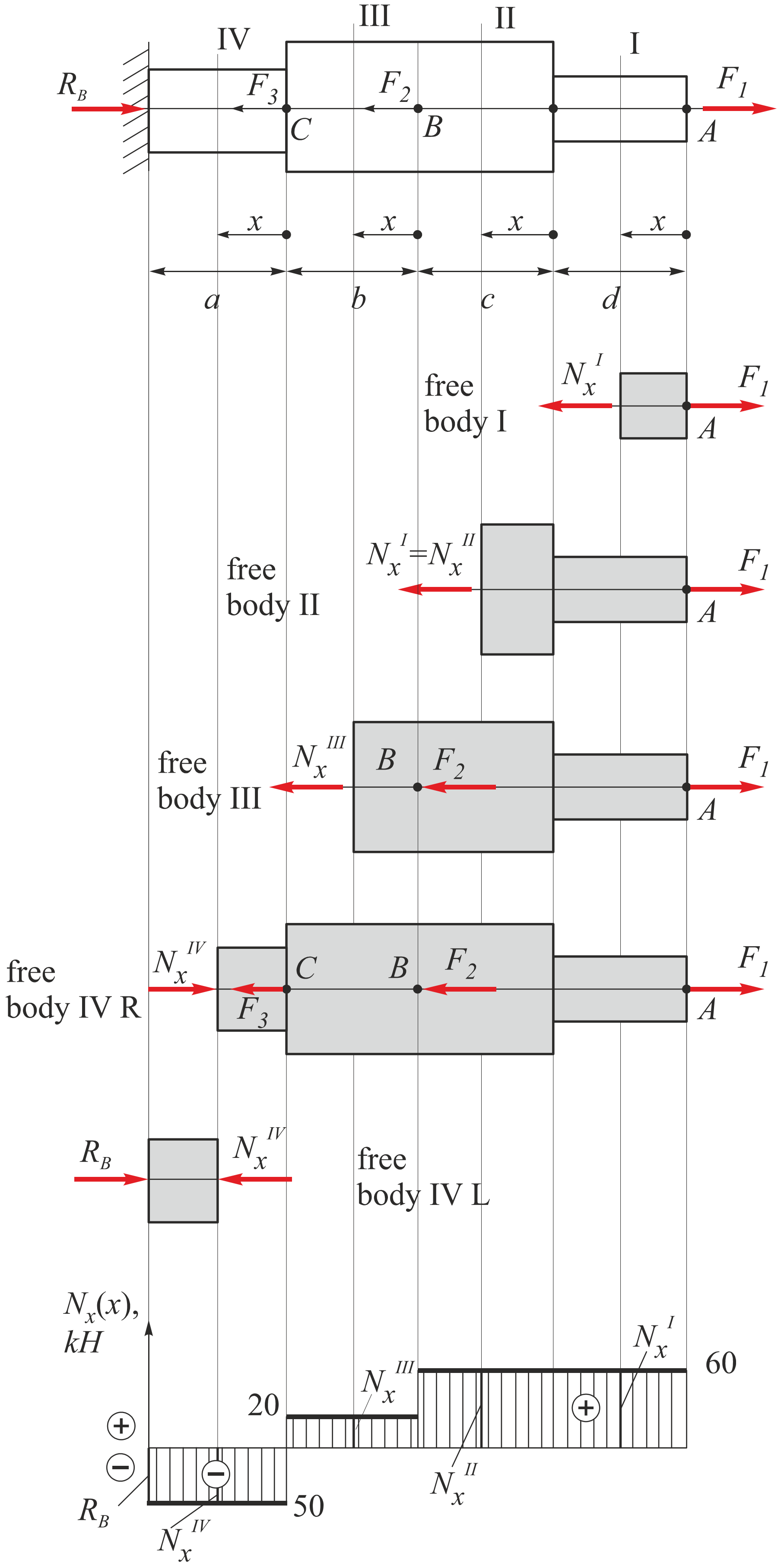
$$N_x^{III}(x) = F_1 - F_2 = 60 - 40 = +20\,\text{kN},$$

$$N_x^{IV}(x) = F_1 - F_2 - F_3 = 60 - 40 - 70 = -50\,\text{kN}.$$

Corresponding free body pictures and diagram $N_x(x) > 0$ are shown on Fig. 6.

One should know the important rule in diagrams of normal forces constructing, since similar rules will be often used in further discussion. ***In points where an external force is applied, including support reactions, a diagram of normal forces shows a "jump" (abrupt) equal in magnitude to applied external force***.

In our example, there are four forces (including the support reaction) and the diagram of normal forces has four jumps equal in magnitude to these forces, respectively (see Fig. 6).



IV
III
II
I
R_B
F_3
F_2
F_1
C
B
A
x
a
b
c
d
free body I
N_x^I
free body II
$N_x^I=N_x^{II}$
free body III
N_x^{III}
free body IV R
N_x^{IV}
free body IV L
$N_x(x)$, kH
20
60
50

Fig. 6

2 Torsional Deformation. Diagrams of Torsional Moments

A bar subjected to torsion is called **shaft**. In shaft torsion, there appears a single internal force factor, **torsional moment**, or **torque**, which acts in the plane of shaft cross section. The examples of torsion deformation are shown in Fig. 7.

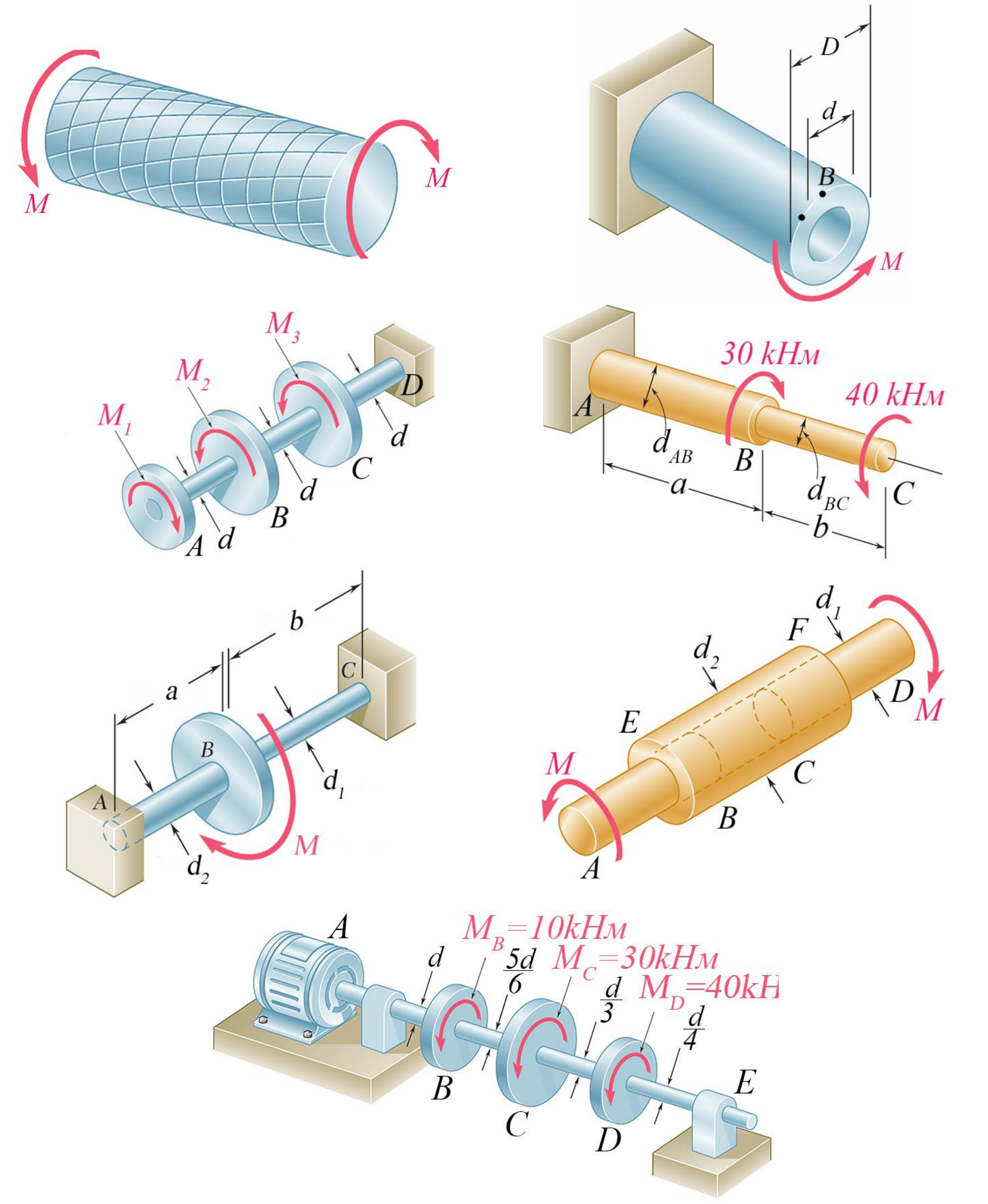


Fig. 7

The internal torque appearing in section can be found from the condition of equilibrium for the right-hand or left-hand portion of the loaded shaft and considering corresponding free body diagram.

Internal torsional moment in the cross-section, as earlier, ***will be equal to the algebraical sum of external moments applied to the part of the shaft under consideration (left or right)***.

The **rule of signs** in torque diagram design: ***if, watching from the tip of a shaft, external torsional moment rotates clockwise, corresponding component of the torque in cross-section of the shaft will be assumed to be positive and vice versa.***

A graph of internal torsional moments distribution along the length of a shaft is called the **torque diagram**.

Example 4 Calculation of internal torque moment M_x in the cross-section of the shaft loaded by concentrated external twisting moment M_T (see page 9).

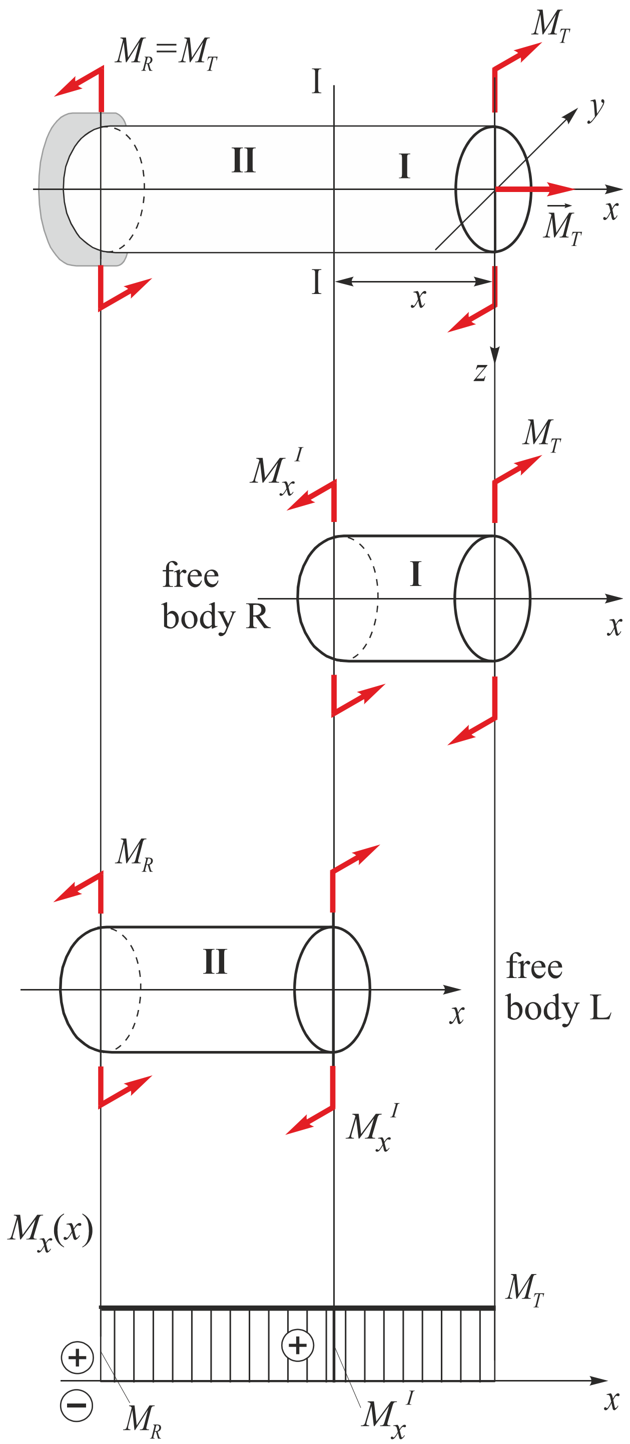


Fig. 8

The condition of equilibrium for the right-hand portion of the twisted shaft is

$$\sum M_x = +M_T - M_x = 0,$$

whence

$$M_x(x) = M_T .$$

The same result corresponds to the equilibrium of left-hand portion of twisted shaft:

$$\sum M_x = +M_x^I - M_R = 0,$$

whence

$$M_x^I = M_R = M_T .$$

In the case when a few external moments application, when writing the equations of internal torsional moments we should apply the sign convention mentioned above: clockwise rotating external moments should be substituted with plus sign and vise versa.

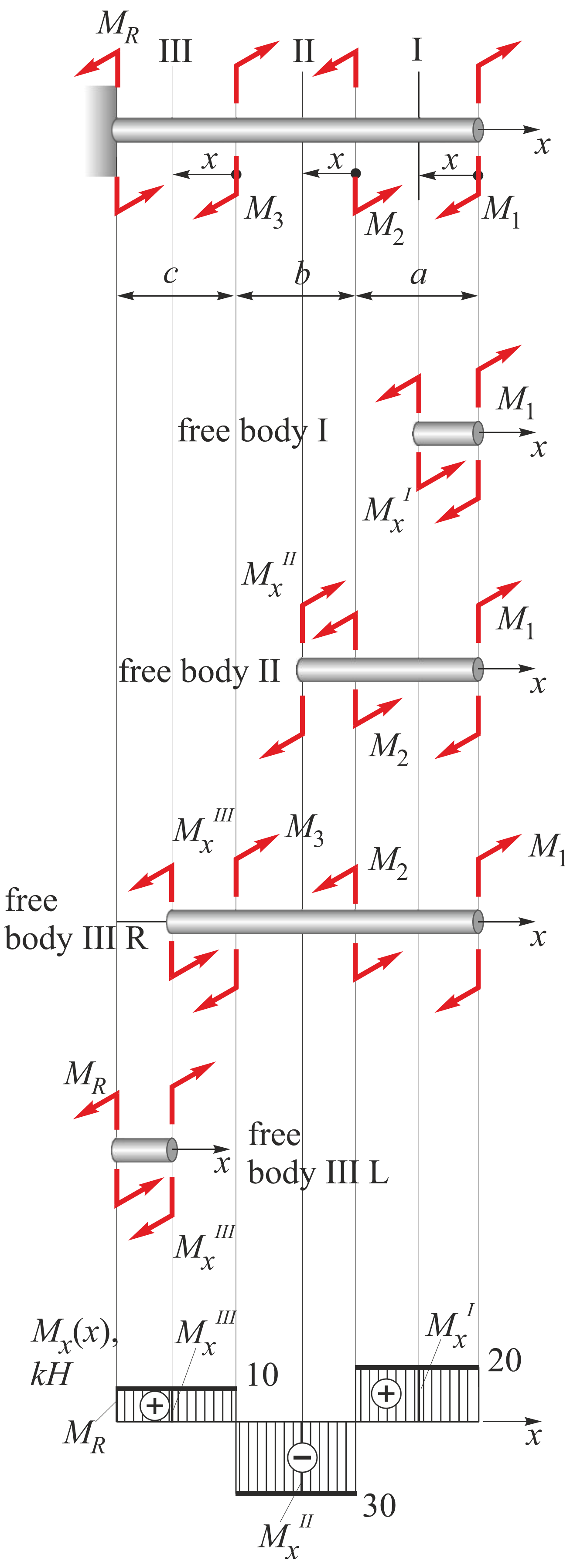


Fig. 9

Example 5 Design of torque moment diagram for the shaft under an arbitrary loading

Given: $M_1 = 20\,\text{kNm}$, $M_2 = 50\,\text{kNm}$, $M_3 = 40\,\text{kNm}$.

R.D.: $M_x(x) = ?$

The internal torsional moment in a particular section of a shaft is equal to the algebraic sum of all external torsional moments which act on the shaft up to the section considered (right situated portions in this case):

$$M_x = \sum_{i=1}^{k} M_x(F_i),$$

I - I: $0 > x > a$

$$M_x^I(x) = M_1 = 20\,\text{kNm},$$

II - II: $0 > x > b$

$$M_x^{II}(x) = M_1 - M_2 = 20 - 50 = -30\,\text{kNm},$$

III - III: $0 > x > c$

$$M_x^{III}(x) = M_1 - M_2 + M_3 = 20 - 50 + 40 = \\ = +10\,\text{kNm}.$$

Let us now construct the diagram of torsional moments. For each portion of the shaft, we lay off M_x on a chosen scale in the same way as has been done for constructing the diagram of normal forces N_x in tension.

A diagram of torsional moments has jumped in points of application of external torsional moments, and jump magnitudes are equal to the magnitude of applied torsional moment.

In the considered case, when moving leftwards along the shaft, we observe the first jump equal to M_1, the second jump equal to M_2, the third jump M_3 and the fourth jump is equal to the reaction moment in the built-in end of the shaft.

MINISTRY OF EDUCATION AND SCIENCE OF UKRAINE

National aerospace university “Kharkiv Aviation Institute”

Department of aircraft strength

Course
Mechanics of materials and structures

HOME PROBLEM 2

Graphs of Normal Force Distribution in Tension-Compression

Name of student:

Group:

Advisor:

Data of submission:

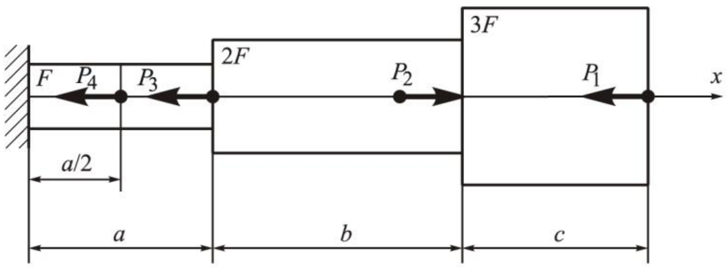
Mark:

National aerospace university
“Kharkiv Aviation Institute”
Department of aircraft strength

Subject: mechanics of materials
Document: home problem
Topic: graphs of normal force distribution in tension-compression of a rod

Full name of the student, group

Variant: 1 **Complexity:** 1



Given: $P_1 = 20$ kN, $P_2 = 40$ kN, $P_3 = 100$ kN, $P_3 = 80$ kN
$a = 3$ m, $b = 4$ m, $c = 5$ m.

Goal: obtain equations of normal force in cross-sections of a rod and draw the graphs of its distribution along the length of a rod.

Full name of the lecturer **signature**

Mark:

In calculating the normal force in the rod cross-sections, we will use the rule that the normal force in the cross-section is numerically equal to algebraic sum of external forces applied to the right or to the left part of the rod. Tensile external force should be substituted into the equation of normal force with positive sign and visa versa. This sign convention is shown on Fig. 1.

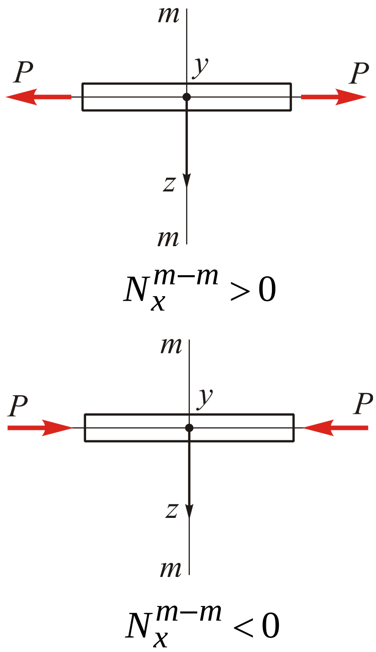


Fig. 1

Solution

1. Selecting the arbitrary cross-sections at x-distances from the origin of each potion.

In this solution, we will consider the equilibrium of right-situated parts of the rod to exclude preliminary calculating the support reaction (see Fig. 2).

2. Writing the equations of normal force in an arbitrary cross-sections of each potion.

I – I $(0 < x < c)$:

$$N_x^I(x) = -P_1 = -20 \text{ kN}.$$

II – II $(0 < x < b)$:

$$N_x^{II}(x) = -P_1 + P_2 = -20 + 40 = 20 \text{ kN}.$$

III – III $(0 < x < a/2)$:

$$N_x^{III}(x) = -P_1 + P_2 - P_3 = -20 + 40 - 100 = -80 \text{ kN}.$$

IV – IV $(0 < x < a/2)$:

$$N_x^{IV}(x) = -P_1 + P_2 - P_3 - P_4 = -20 + 40 - 100 - 80 = -160 \text{ kN}.$$

3. Designing the graph of normal force distribution (see Fig. 2).

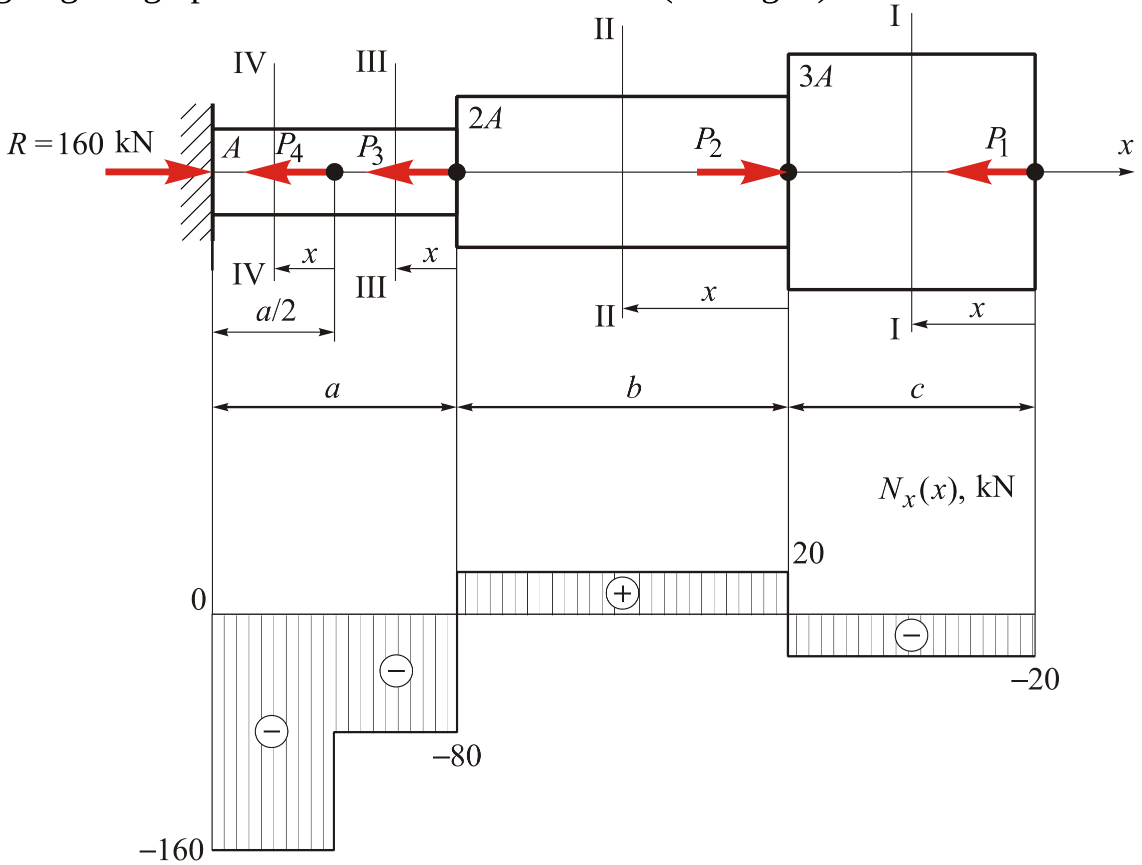


Fig. 2

4. Determining the reaction in support using the graph of normal force distribution.
It is clear from Fig. 2 that in the last potion IV-IV normal force is -160 kN (compression).
The balance of remaining part of the rod under the normal force $N_x^{IV} = -160$ kN from the right and the reaction R from the left shows that R must be equal in modulus to normal force, i.e. $R = 160$ kN and be directed to the right to create negative N_x^{IV}. This fact is shown on Fig. 2.

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HOME PROBLEM 3

Graphs of Torsional Moment Distribution in Torsion

Name of student:

Group:

Advisor:

Data of submission:

Mark:

National aerospace university
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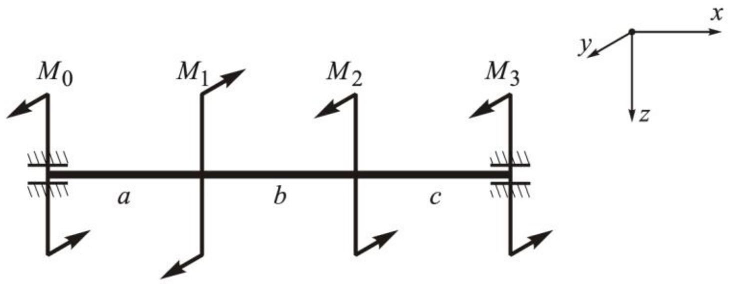
Department of aircraft strength

Subject: mechanics of materials
Document: home problem
Topic: graphs of torsional moment distribution in torsion of a shaft

Full name of the student, group

Variant: 1 **Complexity: 1**



Задано: $M_1 = 10$ kNm, $M_2 = 40$ kNm, $M_3 = 50$ kNm, $a = 2$ m, $b = 3$ m, $c = 1$ m.

Goal: 1) calculate M_0, using condition of a shaft equilibrium; 2) obtain the equations of internal torsional moment in the cross-sections of a shaft and design the graph of its distribution along the shaft length.

Full name of the lecturer **signature**

Mark:

In internal torque moments calculating, we will use the rule that internal torsion moment is equal to the algebraic sum of external moments applied to the right or to the left part of the shaft. We will substitute particular external torsion moment with positive sign into torsion moment equation if it acts clockwise and vice verse. This sign convention is illustrated on Fig. 1.

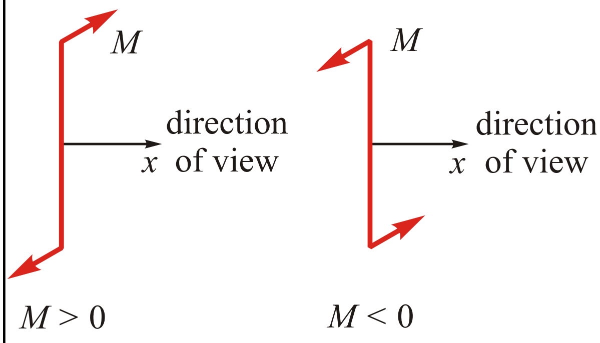


Fig. 1

Solution

1. Calculating unknown M_0 moment applying condition of the shaft equilibrium.

$$\sum M = M_0 + M_1 - M_2 + M_3 = 0 \rightarrow M_0 = -M_1 + M_2 - M_3 = -20 \text{ kNm}.$$

Note, that due to negative M_0 sign its original direction should be changed on opposite (see Fig. 2)

2. Selecting the arbitrary cross-sections at *x*-distances from the origin of each potion. In this solution, we will consider the equilibrium of right-situated parts of the rod (movement from right to left) (see Fig. 2).

3. Writing the equations of internal torque moment in an arbitrary cross-sections of each potion.

$$\text{I} - \text{I} \quad (0 < x < c): \qquad M_x^I(x) = +M_3 = +50 \text{ kNm},$$

$$\text{II} - \text{II} \quad (0 < x < b): \qquad M_x^{II}(x) = +M_3 - M_2 = +50 - 40 = +10 \text{ kNm},$$

$$\text{III} - \text{III} \quad (0 < x < a): \qquad M_x^{III}(x) = +M_3 - M_2 + M_1 = +50 - 40 + 10 = +20 \text{ kNm}.$$

4. Designing the graph of torque moment distribution (see Fig. 2).

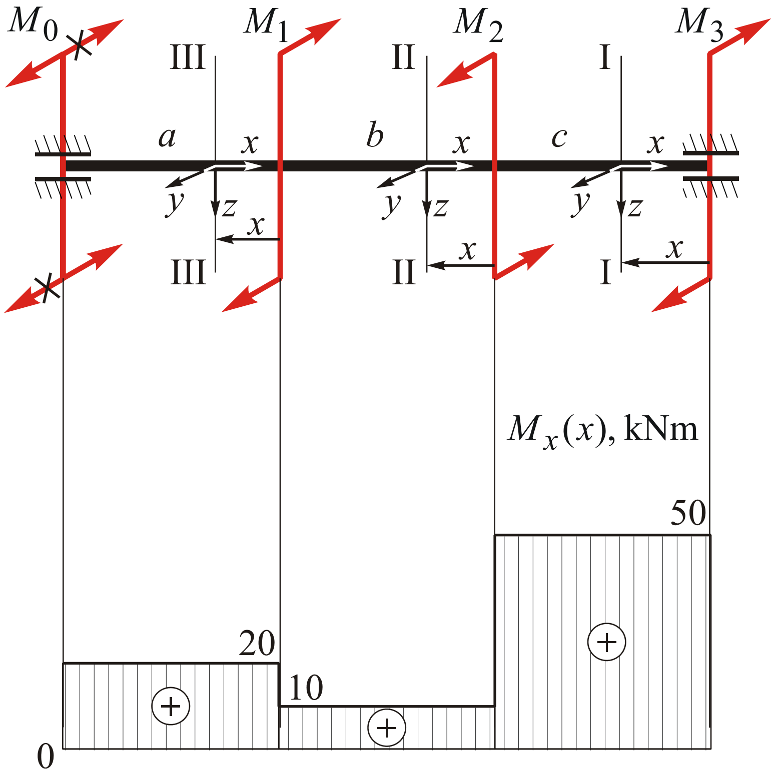


Fig. 2