LECTURE 5 Internal Forces in Beams in Plane Bending Deformation. Diagrams of Internal Forces

Introduction

***A bar subjected to bending is called a* beam**.

Let us consider a model of bending when a plane of bending load coincides with one of cross-sectional principal planes. This type of bending is sometimes called **simple bending**. Actually, it is **plane bending**. The examples of plane bending deformation are shown on Fig. 1.

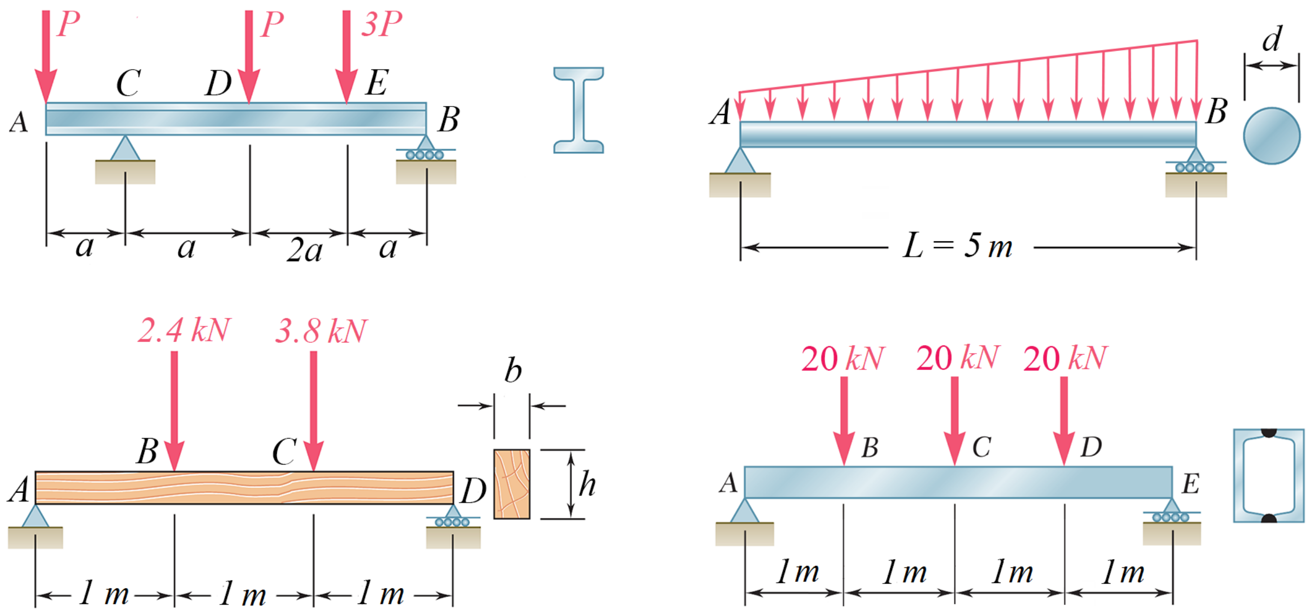


Fig. 1

It is usual to distinguish **pure bending** (Fig. 2) and **transverse bending** (Fig. 3). Pure bending is a type of loading producing the only **bending moment** (M_y or M_z) in cross-section of a rod. In transverse bending, both **shearing force** (Q_z or Q_y) and **bending moment** (M_y or M_z) occur in a rod section. The examples of pure plane bending and transverse plane bending are shown on Figs. 2 and 3, respectively.

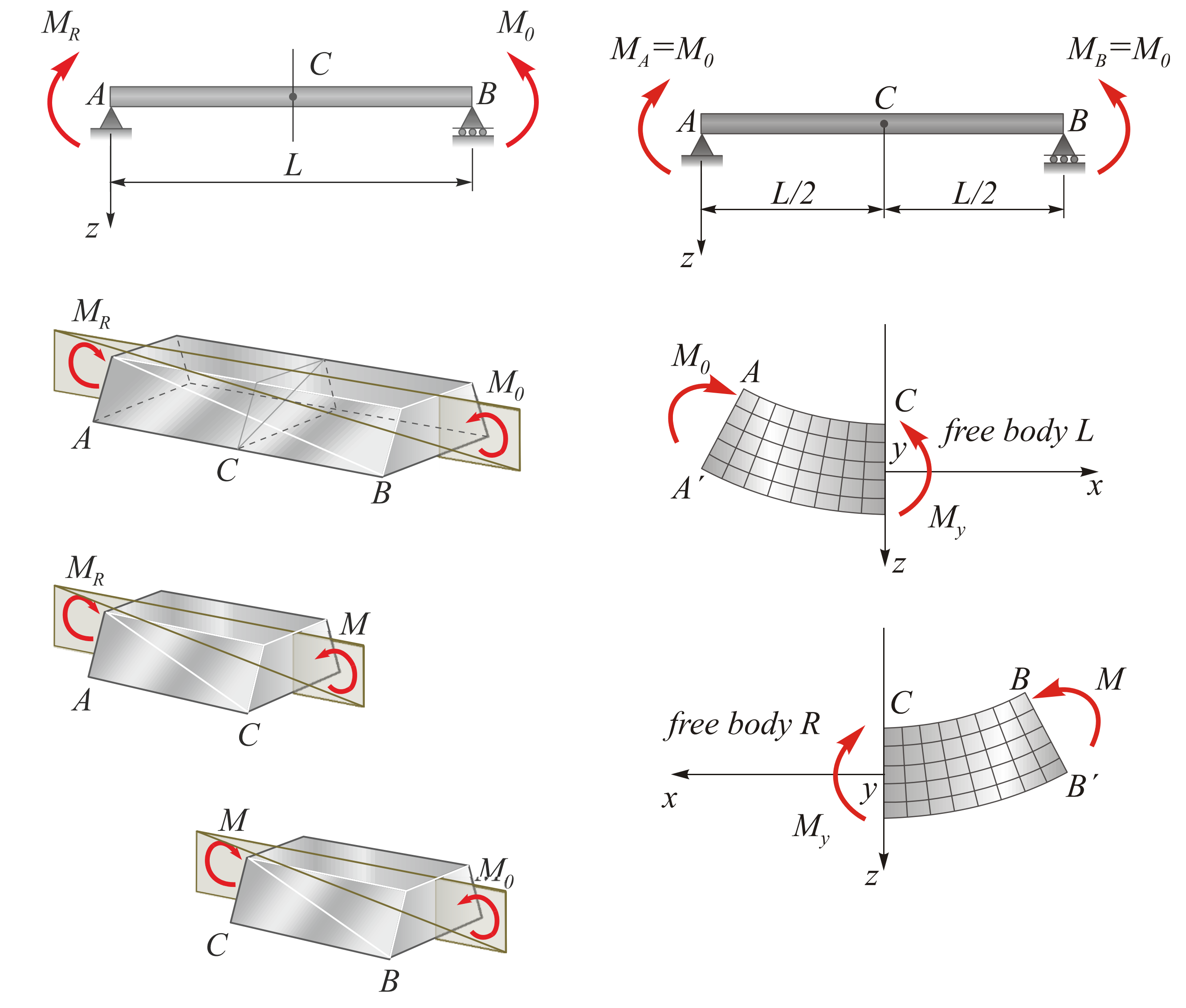


Fig. 2

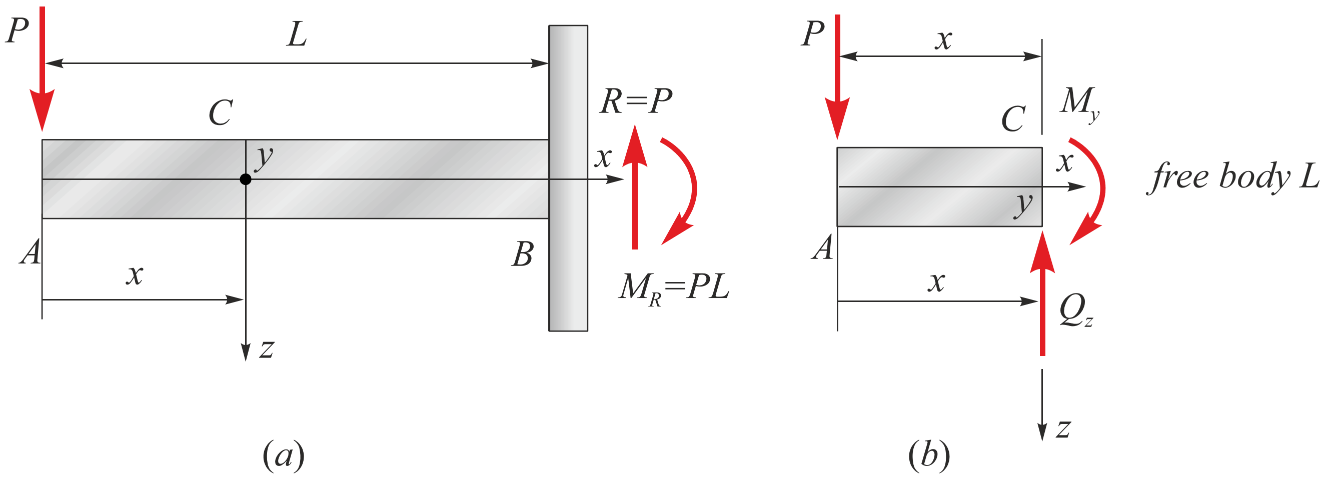


Fig. 3

1 Types of beam supports

There are three main types of beam supports:

1. **Movable hinged (pin) support**;

2. **Immobile hinged support**;

3. **Rigidly fixed (built-in) support**.

The force factors that appear in supports are called **support reactions**.

A. Illustration of movable hinged (pin) support

A **movable hinged support** (Fig. 4) permits free axial displacement of the beam on rollers that's why the only one support reaction appears into it. ***This reaction is directed perpendicularly to support plane***.

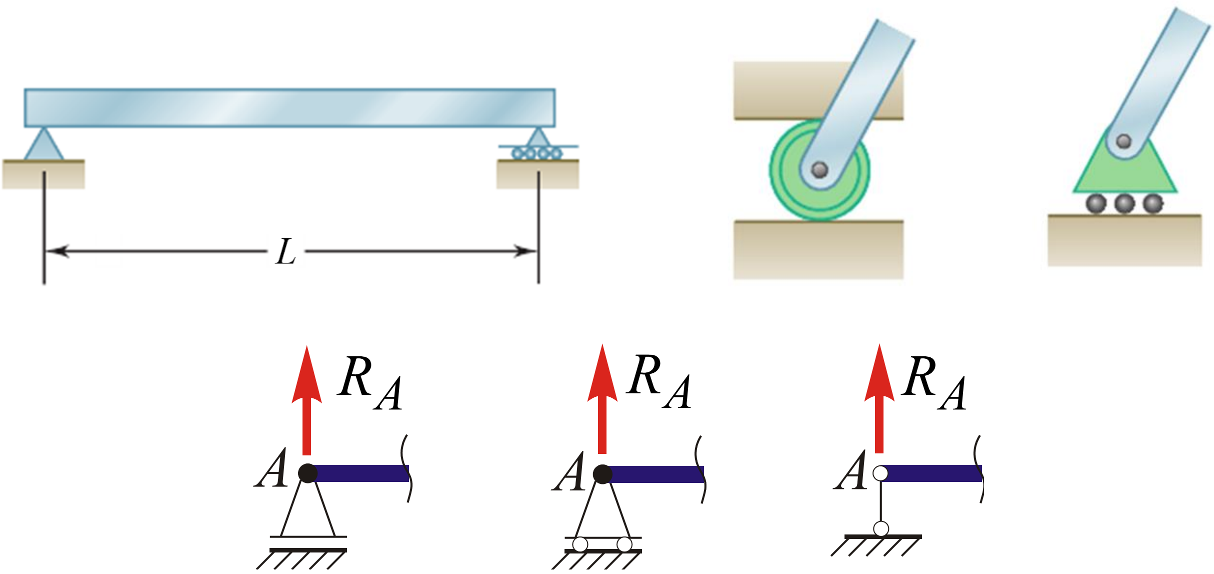
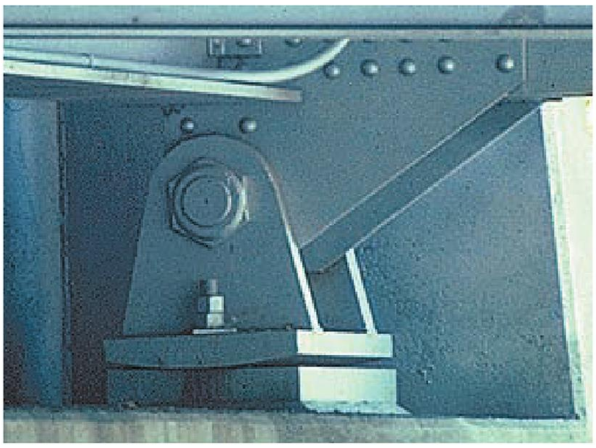
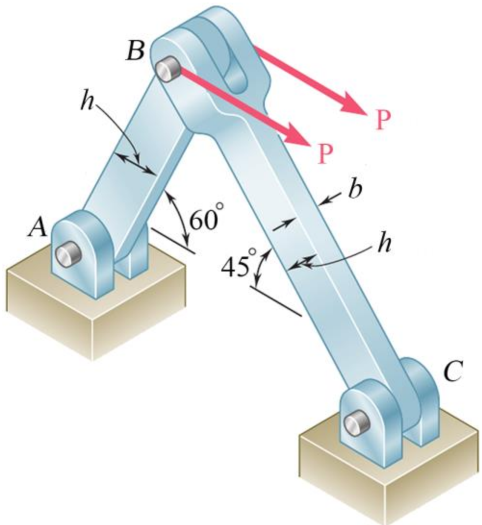


Fig. 4

B. Illustration of immovable hinged (pin) support

At **immobile hinged support**, the reaction is directed to the horizontal axis at an arbitrary angle α; this reaction can be resolved into two reaction components along the horizontal and vertical axes (see Fig. 5).





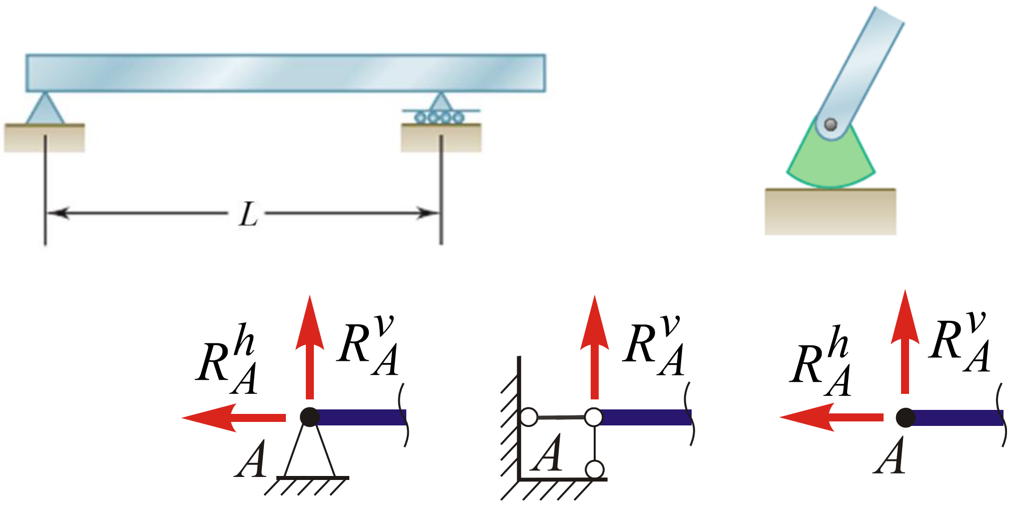


Fig. 5

The first two types of supports permit free rotation of the beam in support cross-section, and therefore, no support moments appear in them (Figs. 4, 5). Only a rigidly fixed (built-in) support, which does not permit the beam cross-section to rotate, can create a reactive support moment (Fig. 6).

C. Illustration of rigidly fixed support

A **rigidly fixed support** gives three reactions: M_R, R^v, R^h (see Fig. 6).
As a rule, the beam is in equilibrium, and therefore, three equilibrium equations are used to describe this balance:

$$\sum F_x = 0, \quad \sum F_z = 0, \quad \sum M = 0. \tag{1}$$

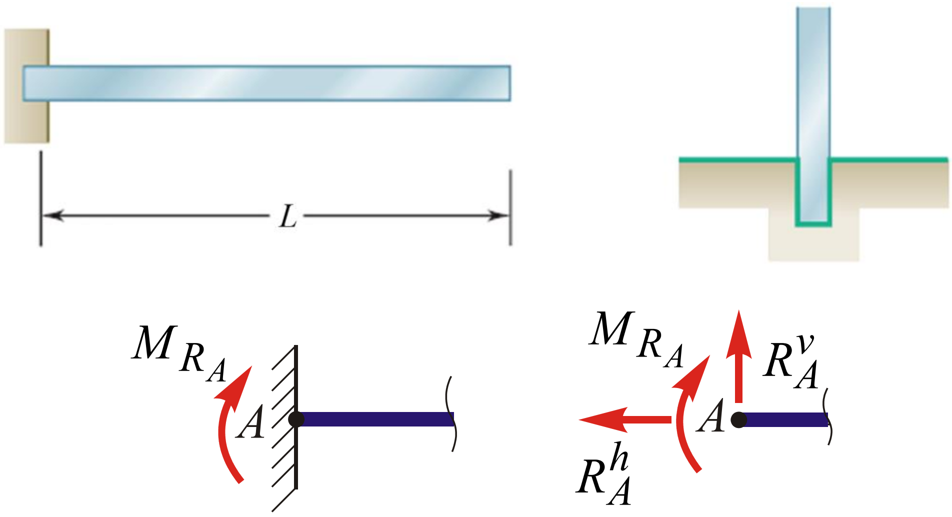
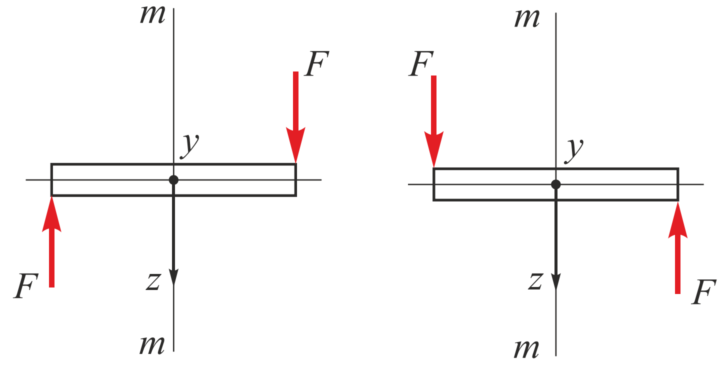


Fig. 6

Equations (1) are used to determine support reactions.

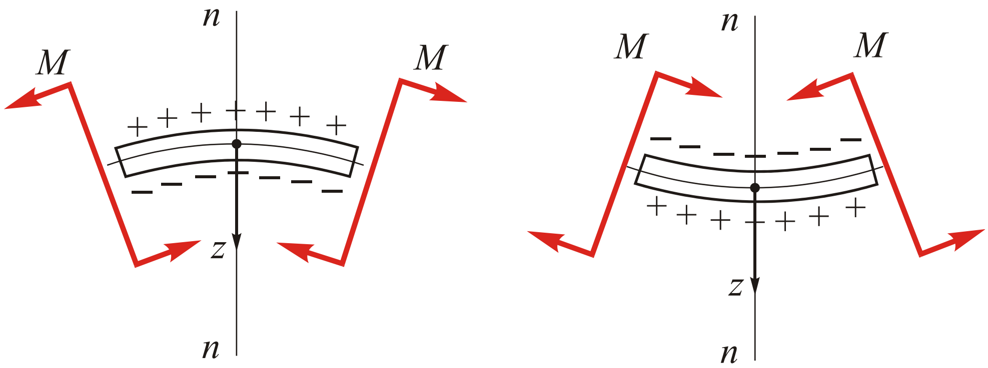
2 The Sign Conventions in Internal Forces Calculation by Method of Sections



$Q_z^{m-m} > 0$ $\qquad$ $Q_z^{m-m} < 0$

Fig. 7

If an external force applied to the left of the section m-m is directed upwards, the shearing force at the section is considered to be positive $(Q_m > 0)$***, otherwise negative*** $(Q_m < 0)$. It means, that the forces tend to produce clockwise rotation (Fig. 7) create $Q_m > 0$; the forces tend to produce counter-clockwise rotation create $Q_m < 0$.



$M_n < 0$ (negative) $\qquad$ $M_n < 0$ (positive)

Fig. 8

The rule of signs for determining the bending moment can be formulated as follows: ***a moment in cross-section n-n is considered to be positive if it affects the beam in the way it becomes convex downwards, and vice versa*** (Fig. 5).

3 Shear Force and Bending Moment Diagrams for Cantilevers

Let us determine the internal force factors in an arbitrary section. To do this, the beam is cut mentally through the section and the unknown internal force factors Q_z and

M_y are applied in the section to produce an equilibrium of any part of the beam under consideration.

The equilibrium equations for the cut-off portion of the beam are as follows:

$$\sum F_z = 0, \quad \sum M_y = 0. \qquad (2)$$

In all cases, the shearing force for a prismatic rod is equal to the algebraical sum of all external forces projections on the plane of section lying on one side of the section (left or right). The bending moment at a section may be regarded as the sum of moments, in relevance to the transverse axis in the section, of all forces applied to one side of this section (left or right). In designing of bending moment and shearing force diagrams with one tip fixed it is possible not to find the reactions in rigidly fixed support. Beams of this kind are commonly called **cantilevers**.

Rule ***We will lay off the values of the bending moments from tensioned fibers of a bent beam.***

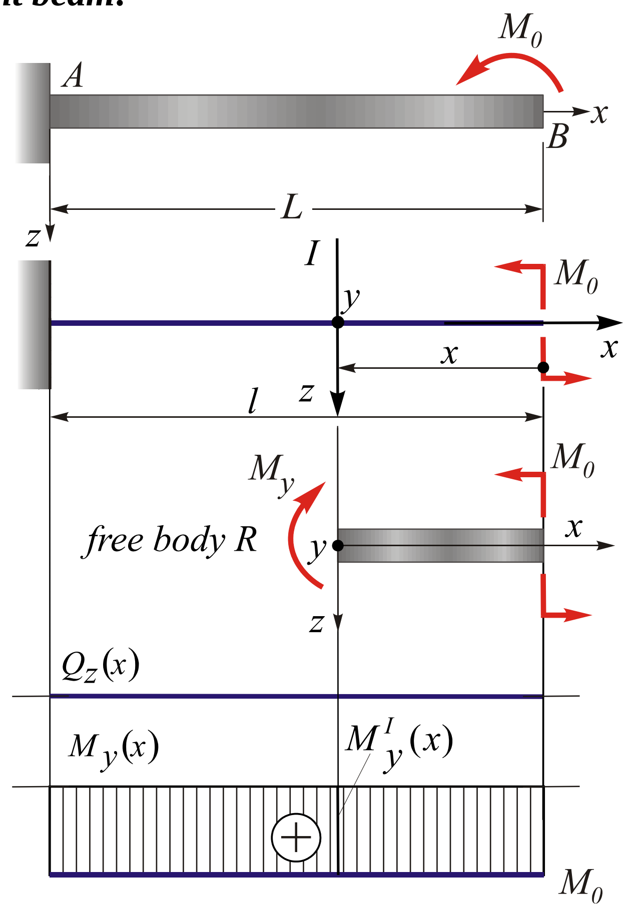


Fig. 9

Example 1 Internal forces induced by concentrated external moment

Given: M, l.

R.D.: $Q_z(x)$, $M_y(x)$.

Equations of internal force factors are:

I-I: $0 < x < l$

$$Q_z^I(x) = 0,$$

$$M_y^I(x) = M.$$

The diagrams are design on Fig. 9.

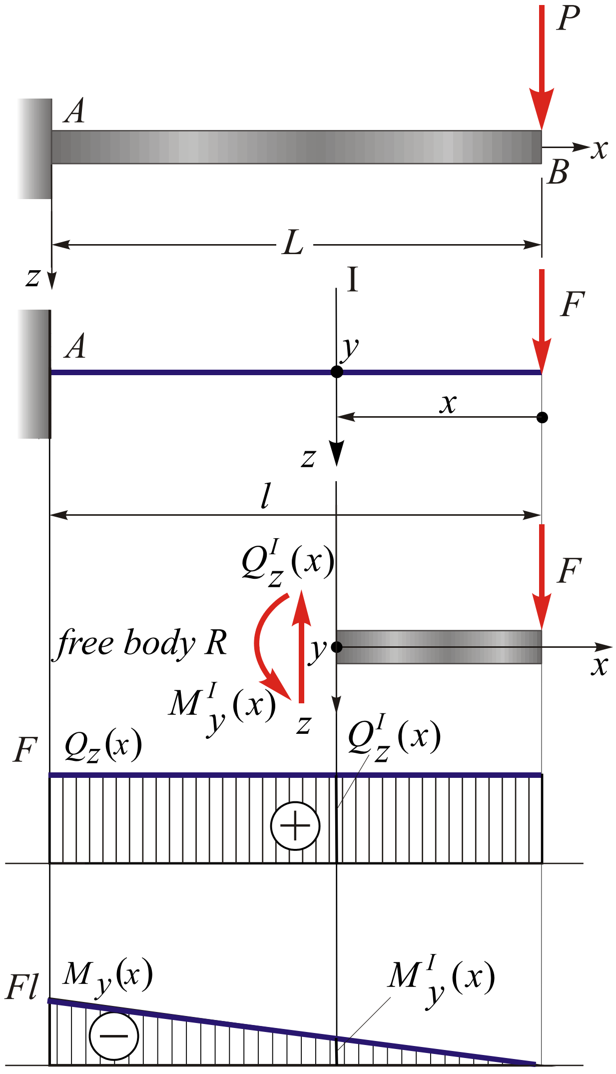


Fig. 10

Example 2 Internal forces induced by concentrated external force

Given: $F, l.$

R.D: $Q_z(x),\ M_y(x).$

Equations of internal force factors are:

I-I: $0 < x < l$

$$Q_z^I(x) = +F\ ,$$

$$M_y^I(x) = -Fx\big|_{x=0} = 0\big|_{x=l} = -Fl\ .$$

The diagrams are design on Fig. 10.

Example 3 Internal forces induced by uniformly distributed external load

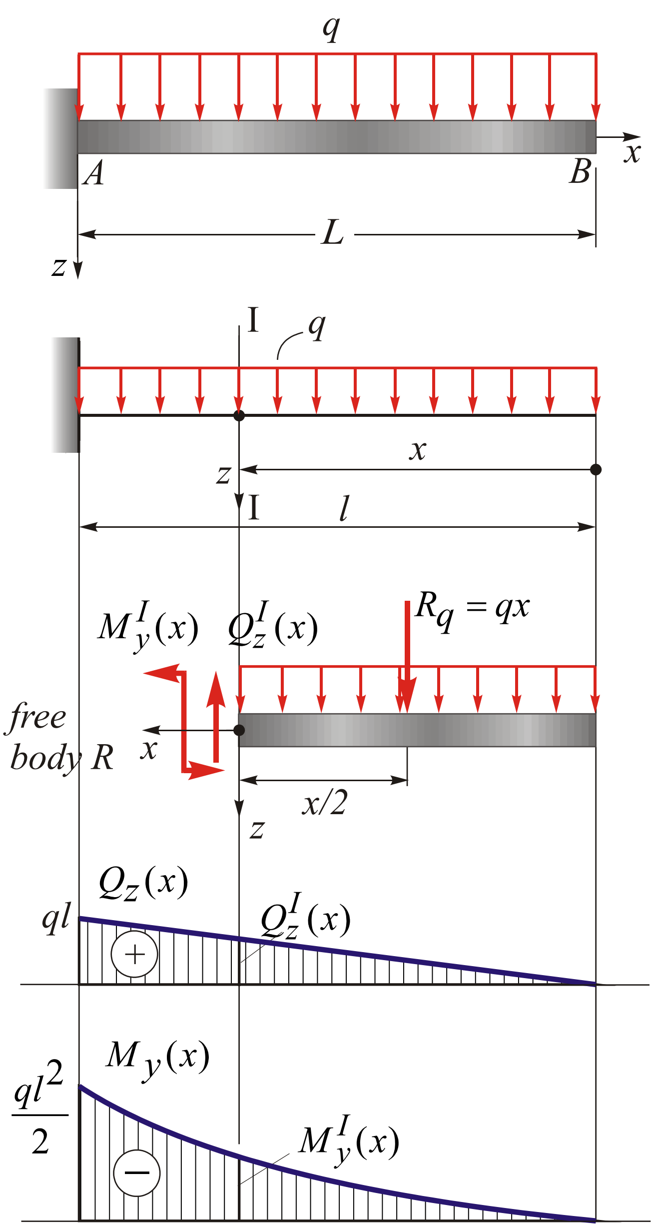


Fig. 11

Given: q, l, where q is the intensity of uniformly distributed loading.

R.D.: $Q_z(x)$, $M_y(x)$.

The **resultant** (R_q) of the uniformly distributed load q is equal to the area of the diagram $q(x)$ and this resultant passes through the "center of gravity" of this diagram.

The shearing force diagram is represented by a straight line. As may be seen from expression (3) the shearing force Q_z in portion I-I increases from 0 to ql ($0 \le x \le l$):

$$Q^I{}_z(x) = +R_q = \\ = qx\Big|_{x=0} = 0\Big|_{x=l} = ql. \quad (3)$$

The maximum value of the shearing force occurs at the built-in support.

The sum of the moments of external forces lying on one (right) side of the section is

$$M^I_y(x) = -R_q\frac{x}{2} = -\frac{qx^2}{2}\Bigg|_{x=0} = 0\Big|_{x=l} = -\frac{ql^2}{2}, \quad (4)$$

where qx is the resultant of uniformly distributed load q of x length. ***This resultant passes through the midpoint of the segment x***. Consequently, the arm of the force is $x/2$, and the moment of this force is equal to $-R_q\frac{x}{2} = -q\frac{x^2}{2}$.

The bending moment diagram is represented by **parabola**. The maximum value of the bending moment occurs at the built-in support. The graphs $Q_z(x)$ and $M_y(x)$ are design on Fig. 11.

Example 4 Internal forces induced by linearly distributed external load with increased intensity

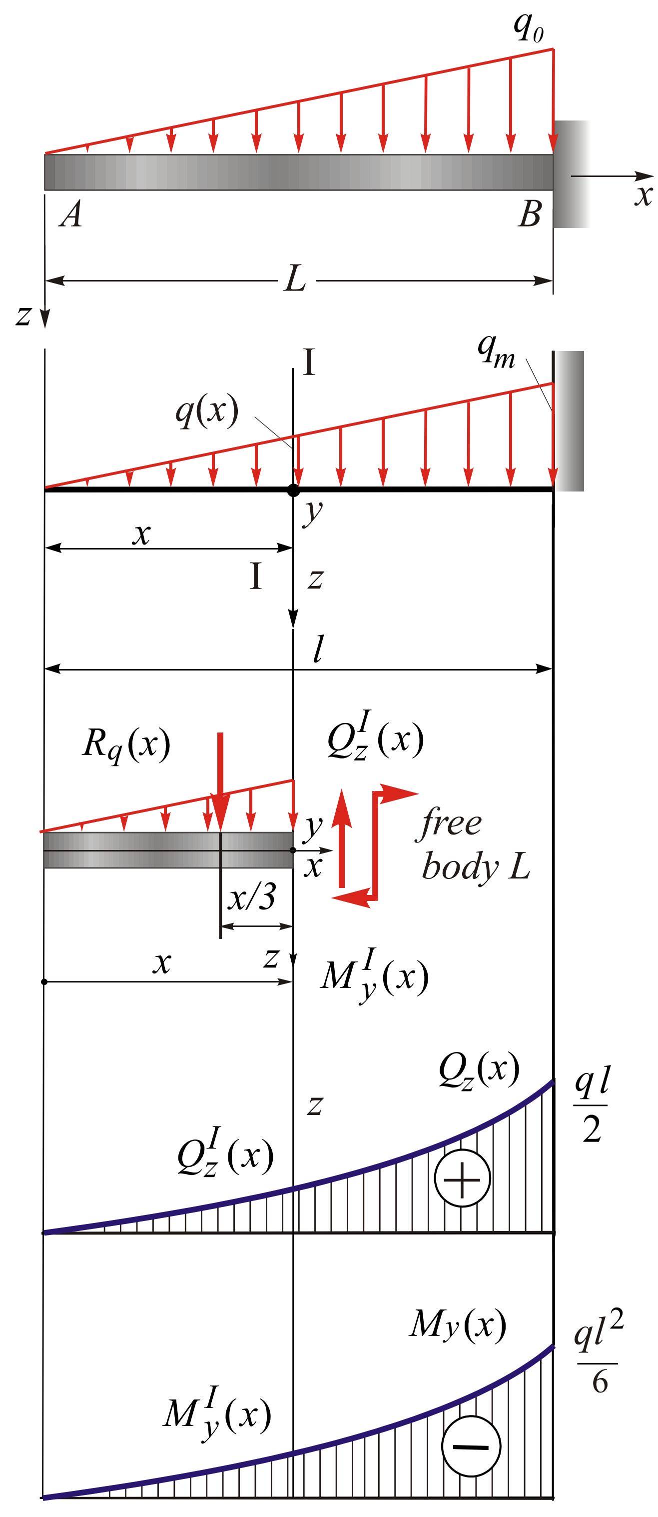


Fig. 12

Given: q_m, l (q_m is maximum intensity of distributed load).

R.D.: $Q_z(x)$, $M_y(x)$.

Let's consider the part of distributed load with the maximum magnitude $q(x)$ applied to the right part of the beam:

Using similarity of the triangles:

$$\frac{q(x)}{q_m}=\frac{x}{l}\rightarrow q(x)=\frac{q_m x}{l},$$

I-I: $0<x<l$

$$Q_y^I(x)=+R_q=+\frac{qx^2}{2l}\bigg|_{x=0}=0\big|_{x=l}=+\frac{ql}{2}$$

$$M_y^I(x)=-R_q\frac{1}{3}x=-\frac{qx^3}{6l}\bigg|_{x=0}=0\big|_{x=l}=$$

$$=-\frac{ql^2}{6}.$$

The graphs $Q_z(x)$ and $M_y(x)$ are design on Fig. 12.

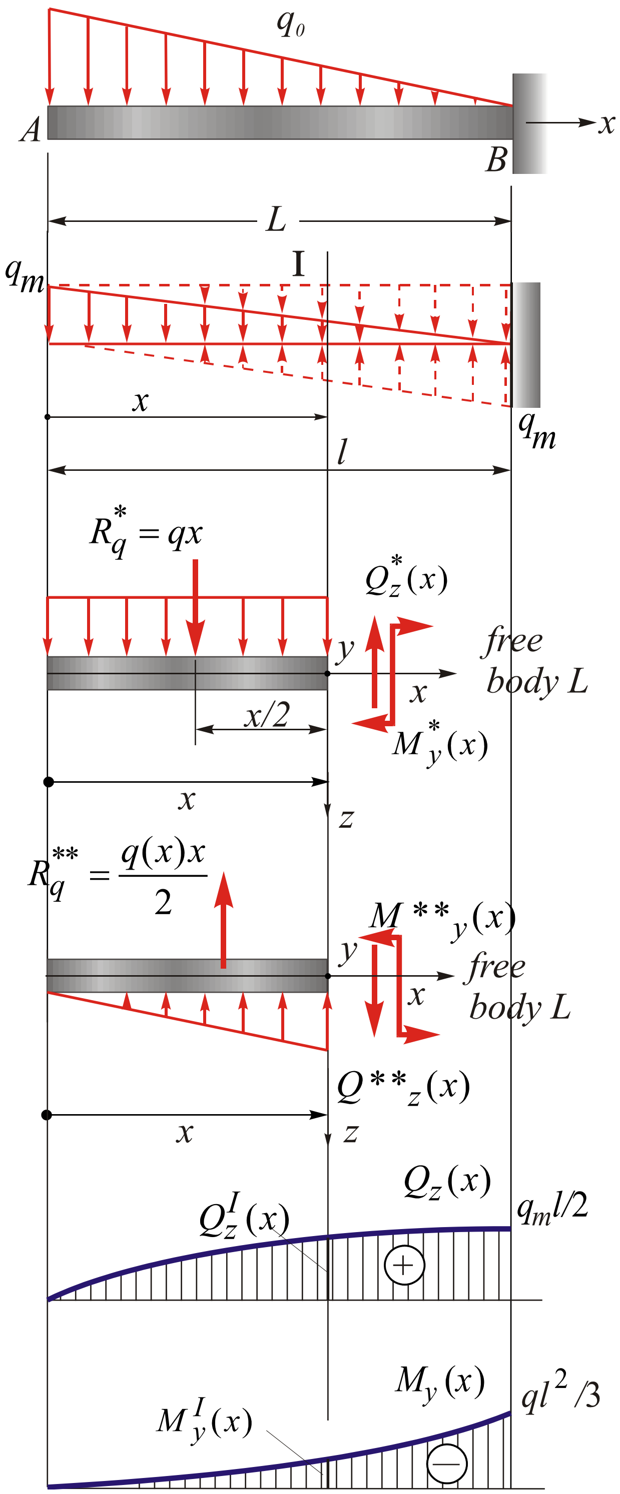


Fig. 13

Example 5 Internal forces created by linearly distributed external load with decreased intensity

Given: q_m, l.

R.D: $Q_z(x)$, $M_y(x)$ – ?

To simplify the solution of this problem let us apply two equal in magnitude but oppositely directed linearly distributed loads of the same intensity. One load is directed downwards and the other is directed upward. Thus, we have static zero in any cross-section. This method alloys to use two previous examples to solve the problem:

I-I: $0 < x < l$

$$Q_y^I(x) = +qx - \frac{qx^2}{2l}\Bigg|_{x=0} = 0\Big|_{x=l} = ql - \frac{ql}{2} = \frac{ql}{2},$$

$$M_y^I(x) = -\frac{qx^2}{2} + \frac{qx^3}{6l}\Bigg|_{x=0} = 0\Big|_{x=l} = -\frac{ql^2}{3}.$$

The graphs $Q_z(x)$ and $M_y(x)$ are design on Fig. 13.

MINISTRY OF EDUCATION AND SCIENCE OF UKRAINE

National aerospace university “Kharkiv Aviation Institute”

Department of aircraft strength

Course
Mechanics of materials and structures

HOME PROBLEM 4

Graphs of Shear Force and Bending Moment Distribution in Plane Bending (Cantilevers)

Name of student:

Group:

Advisor:

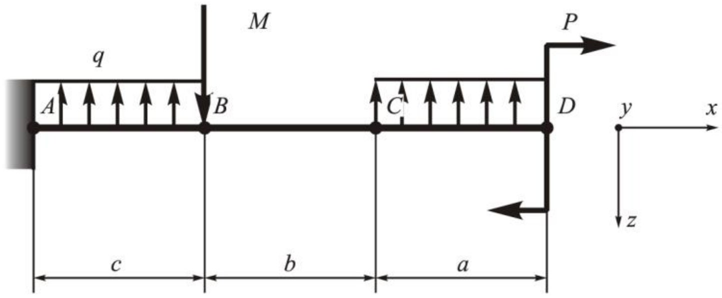
Data of submission:

Mark:

National aerospace university
"Kharkiv Aviation Institute"
Department of aircraft strength

Subject: mechanics of materials
Document: home problem
Topic: graphs of shear force and bending moment distribution along the length of a beam in plane bending deformation.
Full name of the student, group

Variant: 1 **Complexity: 1**



Given: q = 10 kN/m, M = 20 kNm, P = 30 kN, a = 2 m, b = 2 m, c = 2 m.

Goal: obtain the equations of shear force and bending moment in the cross-sections of a beam and design the graphs of their distribution along the beam length.

Full name of the lecturer **signature**

Mark:

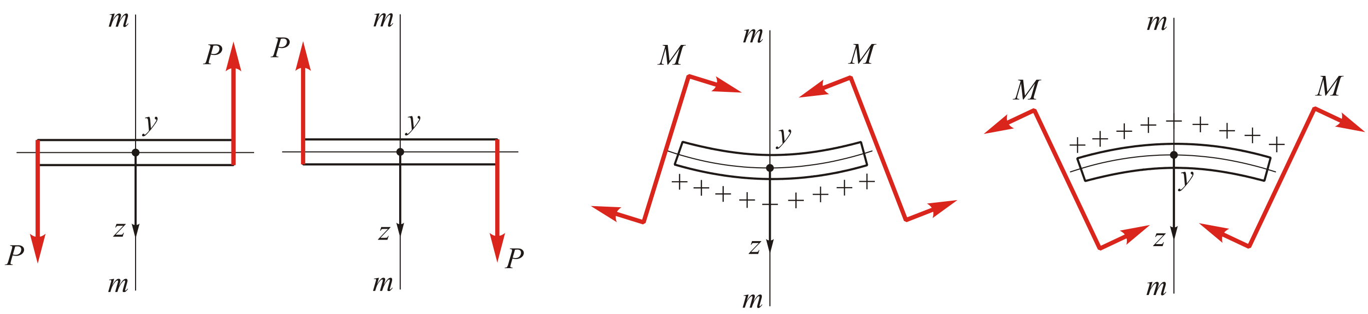
Shear force in a prismatic rod is equal to the algebraic sum of all external forces projections on the cross-section lying on one side of the section (left or right).

The bending moment at a section is equal to the sum of moments, in relevance to the transverse axis in the section, of all external forces applied to one side of the section (left or right).

Solution

1. Accepting the sign conventions in internal forces calculating.

(a) for shear force (b) for bending moment



$Q_z^{m-m} < 0$ $Q_z^{m-m} > 0$ $M_y^{m-m} > 0$ $M_y^{m-m} < 0$

Fig. 1

2. Writing the equations of shear force and bending moment functions in an arbitrary sections of each potion (see Fig. 2).

Note. Method of sections will be applied from the right free tip of the cantilever and the cross-sections will be established at x-distance relative to origin of each potion.

I – I $(0 < x < a)$:

$$Q_z^I(x) = -qx\Big|_{x=0} = 0\Big|_{x=2} = -20 \text{ kN},$$

$$M_y^I(x) = -M + qx\frac{x}{2}\Big|_{x=0} = -20\Big|_{x=2} = 0 \text{ kNm}.$$

II – II $(0 < x < b)$:

$$Q_z^{II}(x) = -qa\Big|_{x=0} = -20\Big|_{x=2} = -20 \text{ kN},$$

$$M_y^{II}(x) = -M + qa\left(\frac{a}{2} + x\right)\Big|_{x=0} = 0\Big|_{x=2} = 40 \text{ kNm}.$$

III – III $(0 < x < c)$:

$$Q_z^{III}(x) = -qa + P - qx\Big|_{x=0} = 10\Big|_{x=2} = -10 \text{ kN},$$

Note, that the change of shear force sign within the boundaries of this section predicts the bending moment extreme value, since the derivative of bending moment is equal to shear force:

$$\frac{d(M_y^{III}(x))}{dx} = qa - P + qx = \left|Q_z^{III}(x)\right|.$$

Therefore, zero shear force and also zero bending moment derivative represent the point of bending moment extreme value.

To find it, let us determine the coordinate of zero shear force x_e and substitute it into bending moment equation.

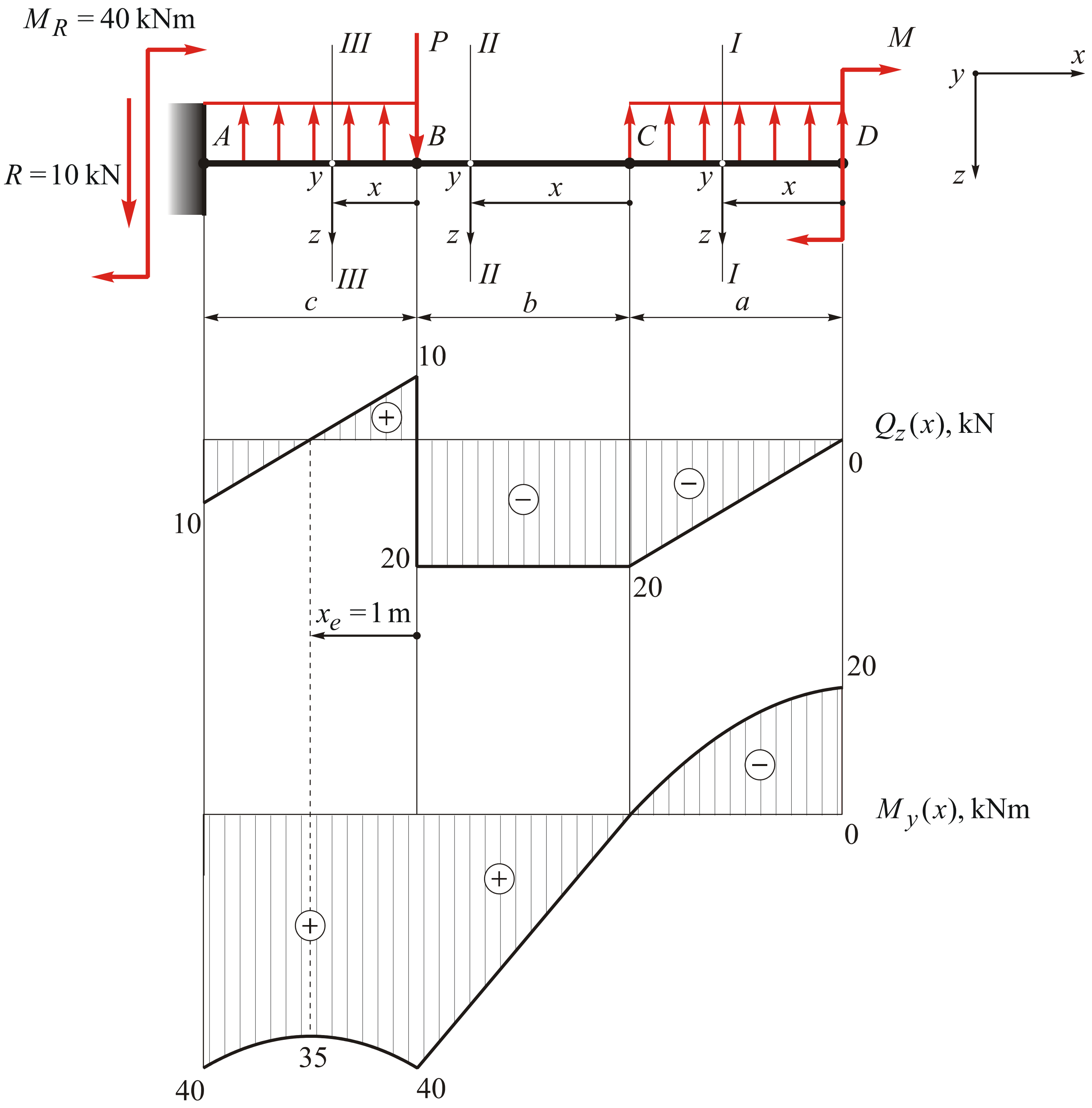
$$Q_z^{III}(x_e) = -qa + P - qx_e = 0 \rightarrow x_e = \frac{1}{q}(P - qa) = \frac{30 - 10\times 2}{10} = 1\,\text{m}.$$

$$M_y^{III}(x) = -M + qa\left(\frac{q}{2} + a + x\right) - Px + qx\frac{x}{2}\Big|_{x=0} = 40\Big|_{x=2} = 40 \text{ kNm}.$$

$$M_y^{III}(x_e) = M_{y_{\max}}^{III} = -M + qa\left(\frac{q}{2} + a + x_e\right) - Px_e + \frac{qx_e^2}{2} = 35 \text{ kNm}.$$

3. Designing the graphs of shear forces and bending moments. Positive shear forces will be drawn upwards and vice versa. Bending moment graph will be drawn on tensile fibers according to the sign convention mentioned above (see Fig. 1 right). The graphs are shown on Fig. 2.

4. Applying the shear force and bending moment values in the rigid support as corresponding reactions according to accepted sign conventions (see Fig. 2).



$M_R = 40$ kNm
$R = 10$ kN
III
P
II
I
M
A
B
C
D
y
x
z
c
b
a
10
$Q_z(x)$, kN
0
10
20
20
$x_e = 1$ m
20
$M_y(x)$, kNm
0
35
40
40

Fig. 2