LECTURE 29 Strength under Cyclic Stresses (Part 2)

1 Effect of Stress Concentration of Fatigue Strength

Numerous theoretical and experimental investigations show that increased stresses are set up in the region of **abrupt changes in shape** of an elastic body as well as in the contact zone of parts.

For example, in a stretched strip with a small **hole** (Fig. 1) the law of uniform stress distribution near the hole is violated.

The state of stress becomes biaxial, and a stress peak appears at the edge of the hole. Similarly, in a stepped bar subjected to bending (Fig. 2) an increased stress occurs in the zone of the **groove**.

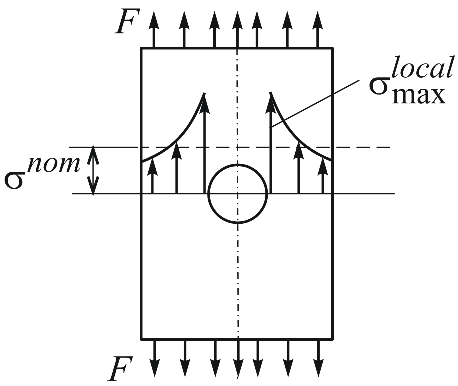


Fig. 1

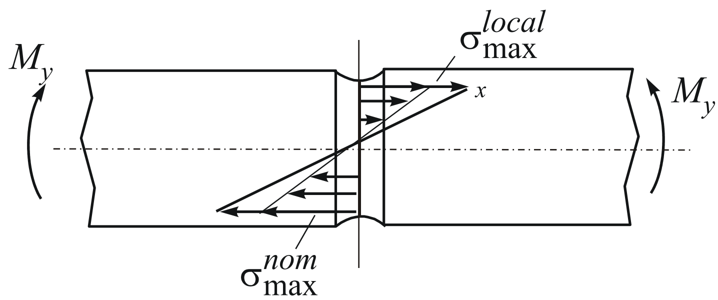


Fig. 2

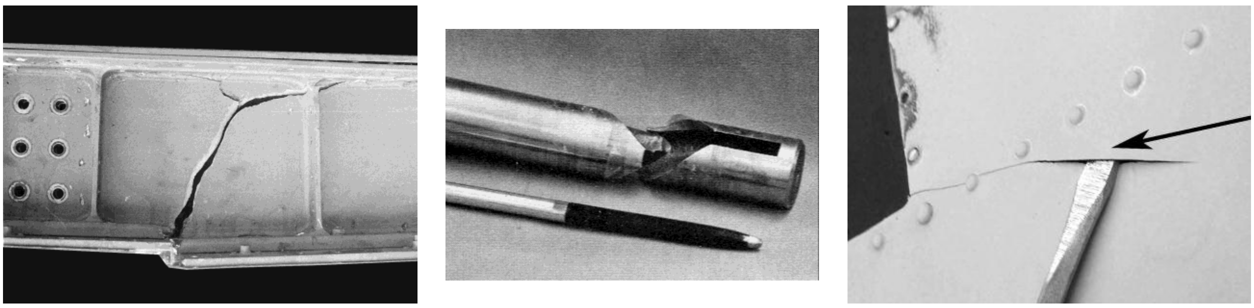


Fig. 3

This phenomenon is called **stress concentration**. Examples of different stress concentrators are shown in Fig. 3. The zone of increased stresses is limited to a narrow region in the vicinity of the focus of stress concentration. Because of the local nature of distribution these stresses are termed localized **(local) stresses**.

The value of localized stresses for different geometrical shapes of parts is usually determined theoretically by the methods of the mathematical theory of elasticity.

The main index of localized stresses is the **theoretical stress concentration factor**

$$\alpha_{\sigma} = \frac{\sigma^{local}{}_{\max}}{\sigma_{nom}}, \tag{1}$$

where $\sigma^{local}_{\max}$ is the maximum **localized stress** and σ_{nom} is the called **nominal stress**. This is the stress which is determined by the formulas of strength of materials with no consideration of the stress concentration effect.

Stress concentration has different effect on the strength of a part depending on the properties of the material and the nature of loading.

Hence the concept of the **effective stress concentration factor** K_{σ} is introduced in addition to the theoretical one, a distinction being made between constant and cyclic stresses. Under cyclic stresses ($r=-1$) the effective stress concentration factor is defined by the ratio

$$K_{\sigma} = \frac{\sigma_{-1}}{\sigma_{-1K}}, \quad K_{\sigma} > 1,$$

where σ_{-1} is the fatigue limit of a **smooth specimen** and σ_{-1K} is the fatigue limit calculated from the nominal stresses for a specimen having stress concentration.

More detailed information on the effective stress concentration factors may be found in the literature (in the handbooks tables).

2 Effect of Surface Finish and Dimensions of a Part on Fatigue Strength

It is apparent that the fatigue limit increases in the case of a clean and well-finished surface.

With **rough finish** the presence of minute surface defects leads to a reduction in fatigue strength indices.

In designing for fatigue strength the properties associated with the surface finish of a part are taken into account by the **surface quality factor**

$$\gamma = \frac{\sigma_{-1\,p}}{\sigma_{-1}}, \quad \gamma > 1 \tag{3}$$

where $\sigma_{-1\,p}$ is the fatigue limit obtained with specimens having **polished surface**; σ_{-1} is the fatigue limit for specimens whose surface finish corresponds to surface finish of the part to be designed.

In the design of a part for fatigue strength it is necessary also to take into account the so-called **size effect** as well as the surface finish effect. Fatigue test made with specimens of various dimensions showed a decrease in the fatigue limit with an increase in the dimensions. Of no little significance is the fact that larger absolute dimensions of a part involve an increase in the probability of structural defects occurring in a region of high localized stresses, with the result that the probability of crack formation increases.

The ratio of the fatigue limit of a part σ_{-1d} to the fatigue limit of specimens of standard dimensions ($d = 8$ to 12 mm) is called the **size effect factor** or simply the size effect

$$\varepsilon_\sigma = \frac{\sigma_{-1d}}{\sigma_{-1}}, \quad \varepsilon_\sigma < 1. \tag{4}$$

3 Factor of Safety in Fatigue and its Analytical and Graphical Determination

Given: σ_{-1}, σ_0, σ_y, K_σ, γ, ε_σ,

where σ_{-1} is the fatigue limit for a completely reversed cycle; σ_0 is the fatigue limit for a pulsating cycle, σ_y is the yield strength. Let us plot the fatigue strength diagram in the system of co-ordinates σ_{av}, $\sigma_{\max}$.

(1) **Limiting symmetrical cycle**:

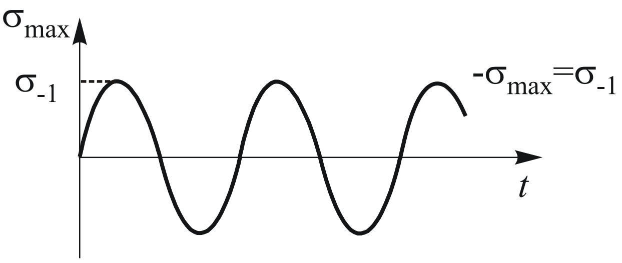


Fig. 4

$$\sigma_{av} = 0$$
$$\sigma_{am} = \sigma_{\max} = \sigma_{-1}$$
$$\sigma_{\max} = \sigma_{av} + \sigma_{am} = \sigma_{-1}$$

(2) **Limiting pulsating cycle**:

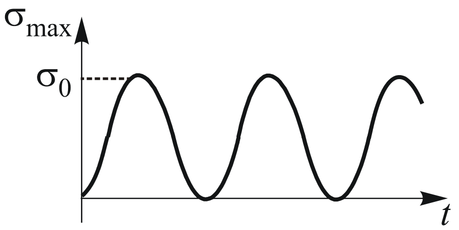


Fig. 5

$$\sigma_{\max} = \sigma_0,$$
$$\sigma_{av} = \frac{\sigma_{\max}}{2} = \frac{\sigma_0}{2},$$
$$\sigma_{\max} = \sigma_{av} + \sigma_{am} = \frac{\sigma_0}{2} + \frac{\sigma_0}{2} = \sigma_0.$$

(3) **Limiting constant cycle**:

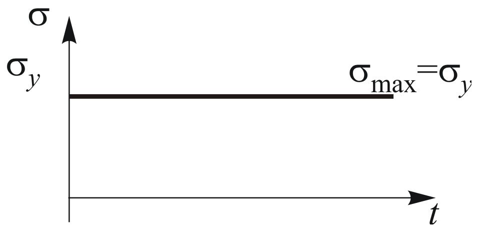


Fig. 6

$$\sigma_{\max} = \sigma_{av} = \sigma_y,$$
$$\sigma_{am} = 0.$$

Fatigue (ultimate) **strength diagram** (Smith's diagram) may be constructed in the coordinates ($\sigma_{\max}$, σ_{av}), using the data obtained from these typical limiting cycles:

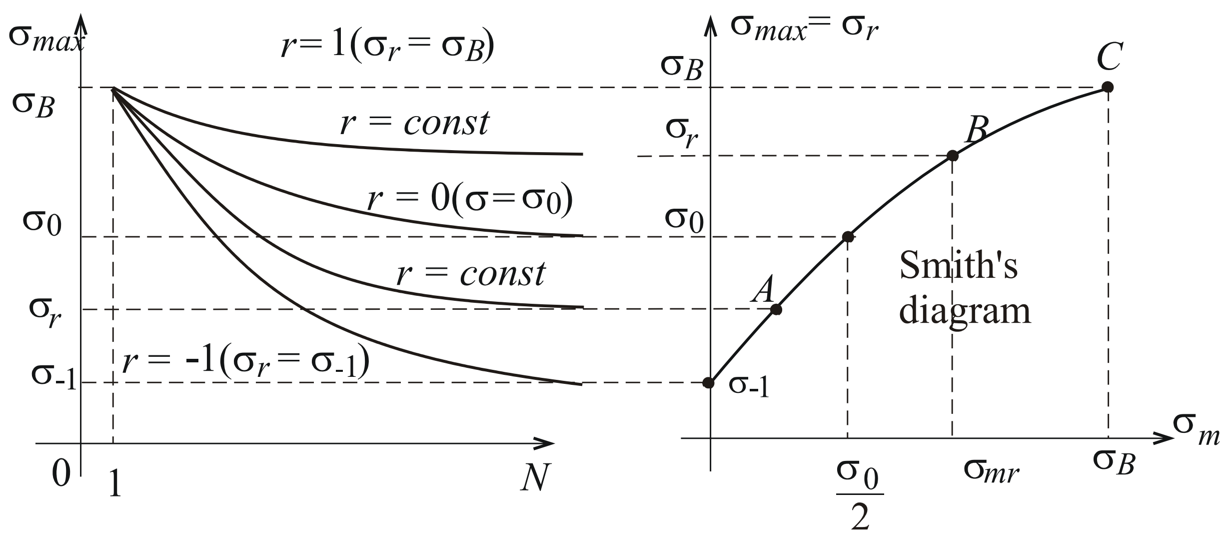
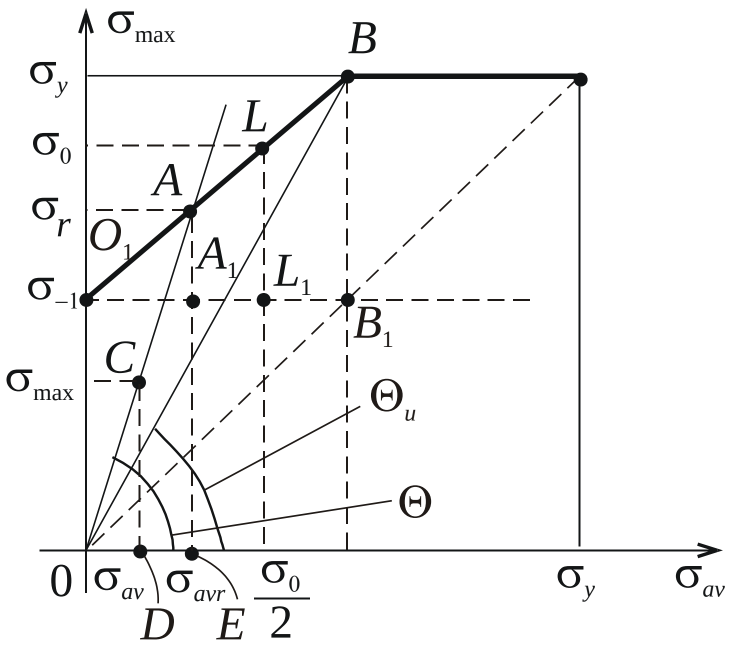


Fig. 7

or in simplified presentation



$$\theta > \theta_u$$

$$\frac{\sigma_{am}^{nom}}{\sigma_{av}^{nom}} \geq \frac{\varepsilon_\sigma}{\gamma K_\sigma} \frac{\sigma_{-1} - \psi_\sigma \sigma_y}{\sigma_y - \sigma_{-1}}.$$

Fig. 8

The diagram is plotted, as shown above, on the basis of given mechanical characteristics of the material σ_{-1}, σ_0 and σ_y.

Suppose for a certain part the cycle is characterized by stress values σ_{av} and σ_{am}. These amounts may by regarded as the co-ordinates of the operating point *C* in the σ_{av}, $\sigma_{\max}$ plane.

The operating point is determined by the nominal values of the cycle stresses σ_{av}^{nom} and σ_{am}^{nom}. Introducing the correction for stress concentration, surface and size effect, the co-ordinates of the operating point take the values $\left(\sigma_{av}^{nom}, \frac{K_\sigma \gamma}{\varepsilon_\sigma} \sigma_{am}^{nom}\right)$.

$$CD = \sigma_{\max} = \sigma_{av}^{nom} + \sigma_{am}^{nom} \frac{K_\sigma \gamma}{\varepsilon_\sigma}. \tag{5}$$

We will consider in future the **factor of safety in fatigue** as the ratio of the segment *AB* to the segment *CD*:

$$n_\sigma = \frac{AE}{CD}. \tag{6}$$

This ratio characterizes the degree of closeness of service conditions to the limiting conditions for a given material.

From geometrical relations between the segments shown in Fig. 6 we obtain

$$AE = \sigma_r = \sigma_{-1} + AA_1 .$$

From the similar triangles O_1LL_1 and O_1AA_1 we find

$$\frac{AA_1}{LL_1} = \frac{O_1A_1}{O_1L_1} \rightarrow AA_1 = \frac{(\sigma_0 - \sigma_{-1})\sigma_{av_r}}{\sigma_0 / 2} = \frac{2(\sigma_0 - \sigma_{-1})}{\sigma_0}\sigma_{av_r} =$$

$$= \left(\frac{\sigma_0 + \sigma_0 - 2\sigma_{-1}}{\sigma_0}\right)\sigma_{av_r} = (1 - \psi_\sigma)\sigma_{av_r} ,$$

where $\psi_\sigma = \dfrac{2\sigma_{-1} - \sigma_0}{\sigma_0}$ is the so-called **sensitivity factor**. The magnitude of ψ_σ depends on the properties of the cycle.

Then

$$AE = \sigma_{-1} + (1 - \psi_\sigma)\sigma_{av_r} . \tag{7}$$

By using expression (6) we obtain

$$n_\sigma = \frac{AE}{CD} = \frac{\sigma_{-1} + (1 - \psi_\sigma)\sigma_{av_r}}{\sigma_{av}^{nom} + \dfrac{K_\sigma \gamma}{\varepsilon_\sigma}\sigma_{am}^{nom}} ;$$

$$\frac{n_\sigma\left(\sigma_{av}^{nom} + \dfrac{K_\sigma \gamma}{\varepsilon_\sigma}\sigma_{am}^{nom}\right)}{\sigma_{av}^{nom}} = \frac{\sigma_{-1} + (1 - \psi_\sigma)\sigma_{av_r}}{\sigma_{av}^{nom}}$$

$$n_\sigma + n_\sigma \frac{K_\sigma \gamma}{\varepsilon_\sigma}\frac{\sigma_{am}^{nom}}{\sigma_{av}^{nom}} = \frac{\sigma_{-1}}{\sigma_{av}^{nom}} + (1 - \psi_\sigma)n_\sigma$$

$$n_\sigma + n_\sigma \frac{K_\sigma \gamma}{\varepsilon_\sigma}\frac{\sigma_{am}^{nom}}{\sigma_{av}^{nom}} = \frac{\sigma_{-1}}{\sigma_{av}^{nom}} + n_\sigma - n_\sigma \psi_\sigma$$

$$n_\sigma\left(\psi_\sigma + \frac{K_\sigma \gamma}{\varepsilon_\sigma}\frac{\sigma_{am}^{nom}}{\sigma_{av}^{nom}}\right) = \frac{\sigma_{-1}}{\sigma_{av}^{nom}} \rightarrow n_\sigma = \frac{\sigma_{-1}}{\sigma_{av}^{nom}\psi_\sigma + \dfrac{K_\sigma \gamma}{\varepsilon_\sigma}\sigma_{am}^{nom}} . \tag{8}$$

This expression is true if $\theta > \theta_u$.

If the amount of θ is less than the θ_u, then

$$n_\sigma = \frac{\sigma_y}{\sigma_{av}^{nom} + \sigma_{am}^{nom}} = \frac{\sigma_y}{\sigma_{\max}},$$

where $\sigma_{\max}$ is the maximum stresses of the cycle.

In exactly the same manner the relations of fatigue strength may by derived for pure shear (torsion). In this case

$$n_\tau = \frac{\tau_{-1}}{\tau_{av}^{nom} + \frac{K_\tau \gamma}{\varepsilon_\tau} \tau_{am}^{nom}}.$$

For biaxial stress-state

$$n = \frac{n_\sigma n_\tau}{\sqrt{n_\sigma^2 + n_\tau^2}},$$

where *n* is the desired factor of safety in fatigue, n_σ is the factor of safety based on the assumption that shearing stresses τ are absent, n_τ is the factor of safety based on shearing stresses, i.e. on the assumption that $\sigma = 0$.