LECTURE 30 Impact Loading

By an impact load is meant, in general, any rapidly varying load. The examples of impact loading in machinery are shown in Fig. 1, where (a) – impact load – moving mass case, (b) – impact loading of a uniform rod.

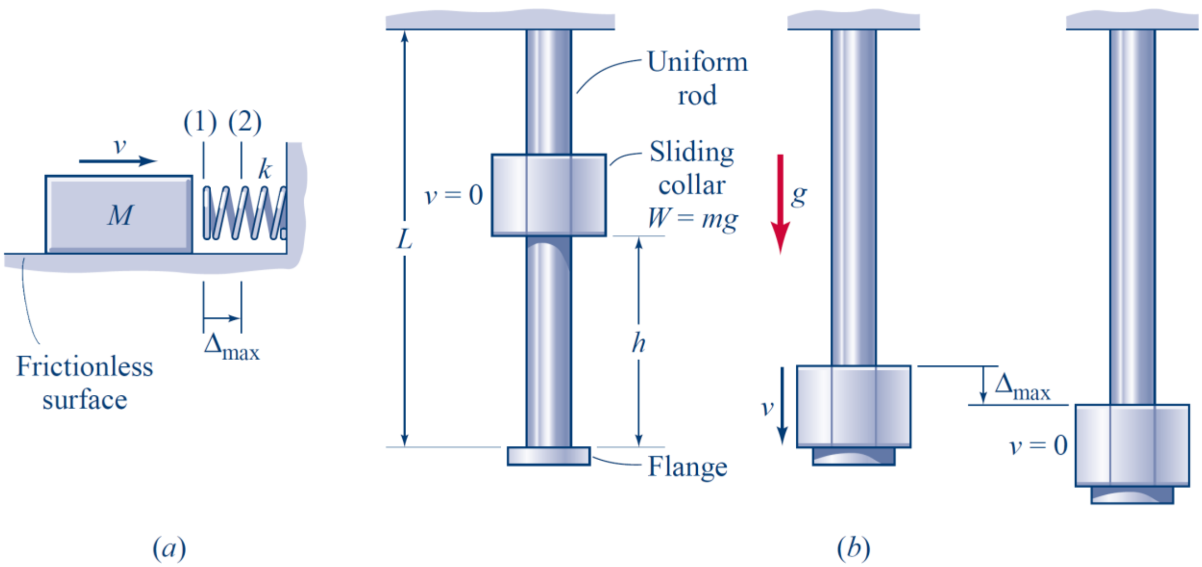

Fig. 1

The problem of structural design for impact loads involves many difficulties which cannot always be overcome by simple means. We shall limit ourselves to the simplest methods of calculation which do not provide high accuracy but permit a correct estimate to be made of the order of displacements, stresses and strains under impact loading. The energy approach is preferable where only the maximum values of dynamic forces and dynamic deflections are to be obtained, but the determination of the laws of motion of the masses is not required. This is just the case in practical calculations.

1 Determination of dynamic factor

Let us consider the vertical motion of a striking weight Q. The weight Q is stopped by the elastic element.

When the total kinetic energy of the weight is transformed into the potential energy of the compressed bar the weight comes to rest. Further the reverse motion starts. But we don't consider the reverse motion.

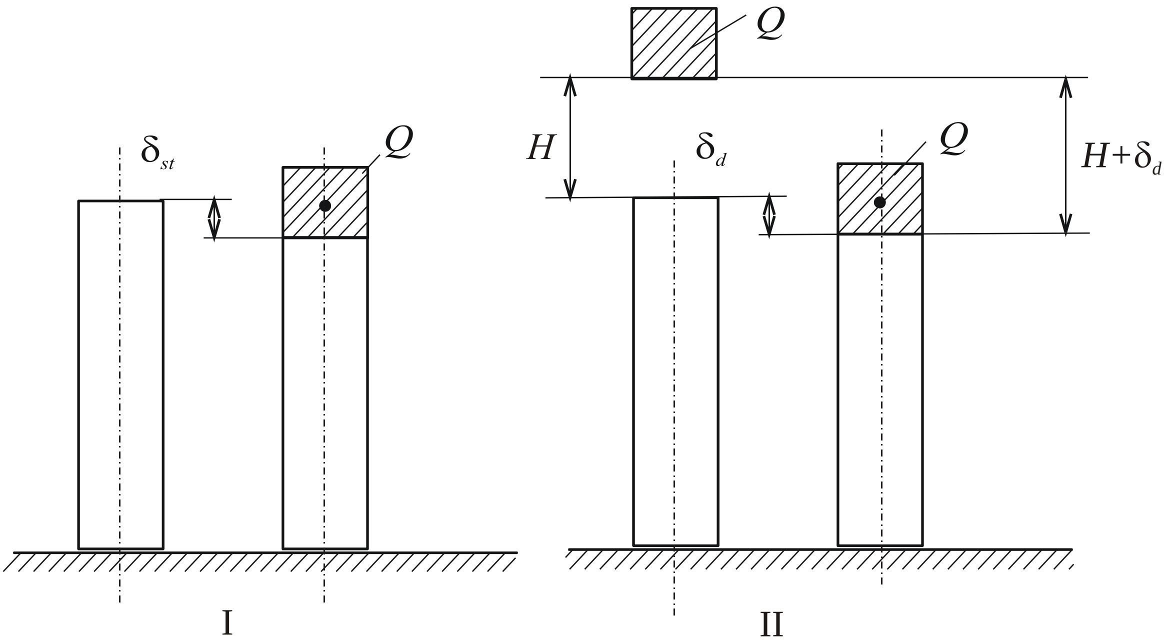

Fig. 2

In setting up the energy balance it is necessary here to take into account the linear relationship between stresses and strains. ***The modulus E in the impact loading is assumed to be equal to the modulus E in the static loading:***

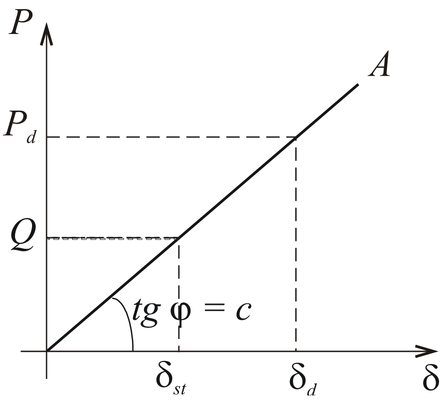

Fig. 3

$$Q = c \times \delta_{st}, \quad (1)$$

$$P_d = c \times \delta_d, \quad (2)$$

$$\delta_d > \delta_{st}, \quad (3)$$

$$K_d = \delta_d / \delta_{st}. \quad (4)$$

The quantity K_d is called the **dynamic factor**. ***The dynamic factor is the ratio between the deflection at impact and deflection produced by the load statically applied.*** The internal forces and the stresses change in the same ratio

$$\sigma_d = K_d \sigma_{st}. \quad (5)$$

Determine the potential energy of the falling weight

$$W_p = Q(H + \delta_d). \quad (6)$$

Determine the strain energy of the elastic body

$$U_p = \frac{1}{2} P_d \delta_d . \quad (7)$$

Because the energy of the falling weight is equal to the potential energy of strain of the elastic rod we can write down

$$Q\left(H + \delta_d\right) = \frac{1}{2} P_d \delta_d . \quad (8)$$

According to expression (1) and (2) we have got

$$c\delta_{st}\left(H + \delta_d\right) = \frac{1}{2} c\delta_d^2 ,$$

$$\delta_d^2 - 2\delta_{st}\delta_d + 2H\delta_{st} = 0 ,$$

whence

$$\delta_d = \delta_{st} + \sqrt{\delta_{st}^2 + 2\delta_{st} H} , \quad \delta_d > \delta_{st}$$

or

$$\frac{\delta_d}{\delta_{st}} = 1 + \sqrt{1 + 2H / \delta_{st}} = K_d .$$

Consequently

$$K_d = 1 + \sqrt{1 + \frac{2H}{\delta_{st}}} \quad (9)$$

where H is the height of the weight fall, δ_{st} is the deflection produced by the statically applied load.

If we know the velocity v in impact $\left(H = v^2 / 2g\right)$, then

$$K_d = 1 + \sqrt{1 + \frac{v^2}{g\delta_{st}}} , \quad (10)$$

where g is the **acceleration of gravity**.

2 Examples

Example 1 Longitudinal impact

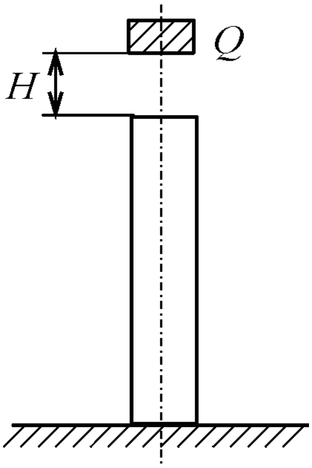

Fig. 4

Given: $Q = 100\,\text{N}$, $H = 4\times10^{-2}\,\text{m}$, $l = 1\,\text{m}$, $A = 10\times10^{-4}\,\text{m}^2$

$E = 2\times10^{5}\,\text{MPa}$

Goal: σ_d – dynamic stress in the rod.

1. $\sigma_{st} = \dfrac{Q}{A} = \dfrac{100N}{10\times10^{-4}} = 0.1\text{MPa}$

2. $\delta_{st} = \dfrac{Pl}{EA} = \dfrac{Ql}{EA} = 0.5\times10^{-4}\,\text{m}.$

3. $K_d = 1 + \sqrt{1 + 2H/\delta_{st}} = 401.$

4. $\sigma_d = K_d \sigma_{st} = 401\times0.1 = 40.1\,\text{MPa}.$

Example 2 Transverse impact

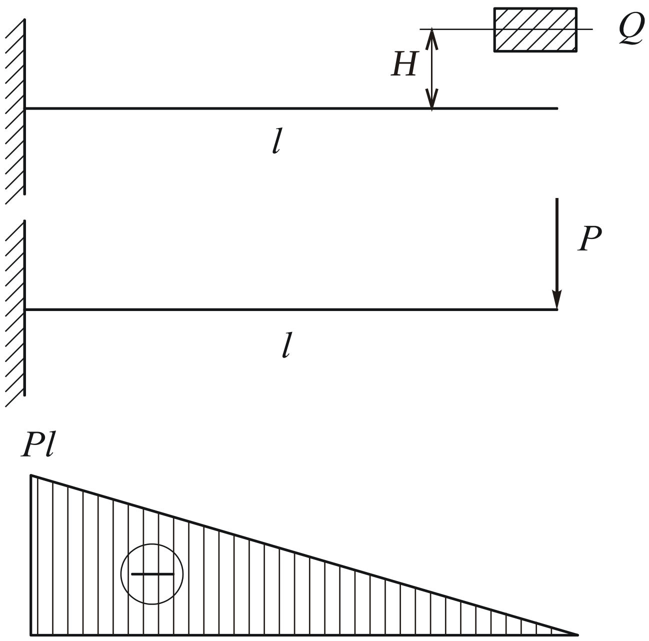

Fig. 5

Given: $l = 100\times10^{-2}\,\text{m}$, $Q = 50\,\text{N}$,

I No10, $W_y = 39.7\times10^{-6}\,\text{m}^3$,

$I_y = 198\times10^{-8}\,\text{m}^4$, $H = 4\times10^{-2}\,\text{m}.$

Goal: σ_d - ?

1. $\sigma_{st\,\max} = \dfrac{M_{y\max}}{W_y} =$

$= \dfrac{50\cdot1}{39.7\cdot10^{-6}} = 1.26\,\text{MPa}.$

2. $\delta_{st} = \dfrac{Pl^3}{3EI_y} =$

$= \dfrac{50\times1}{3\times2\times10^{11}\times198\times10^{-8}} = 4.2\times10^{-5}\,\text{m}.$

3. $K_d = 1 + \sqrt{1 + \dfrac{2H}{\delta_{st}}} = 1 + \sqrt{1 + \dfrac{2\times4\times10^{-2}}{4.2\times10^{-5}}} = 44.6.$

4. $\sigma_d = 1.26\times10^{6}\times44.6 = 56.3\,\text{MPa}.$