MINISTRY OF EDUCATION AND SCIENCE OF UKRAINE

National aerospace university “Kharkiv Aviation Institute”

Department of aircraft strength

Course
Mechanics of materials and structures

# HOME PROBLEM 4

## Graphs of Shear Force and Bending Moment Distribution in Plane Bending (Cantilevers)

Name of student:

Group:

Advisor:

Data of submission:

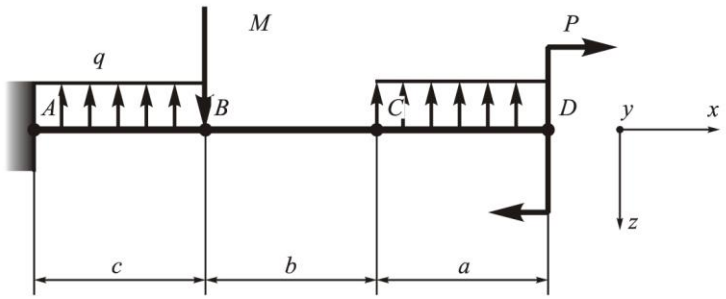
Mark:

**National aerospace university**
**"Kharkiv Aviation Institute"**
**Department of aircraft strength**

**Subject:** mechanics of materials
**Document:** home problem
**Topic:** graphs of shear force and bending moment distribution along the length of a beam in plane bending deformation.
**Full name of the student, group**

---

**Variant: 1** **Complexity: 1**




**Given:** $q = 10$ kN/m, $M = 20$ kNm, $P = 30$ kN, $a = 2$ m, $b = 2$ m, $c = 2$ m.

**Goal:** obtain the equations of shear force and bending moment in the cross-sections of a beam and design the graphs of their distribution along the beam length.

**Full name of the lecturer** **signature**

---

**Mark:**

Shear force in a prismatic rod is equal to the algebraic sum of all external forces projections on the cross-section lying on one side of the section (left or right).

The bending moment at a section is equal to the sum of moments, in relevance to the transverse axis in the section, of all external forces applied to one side of the section (left or right).

## Solution

1. Accepting the sign conventions in internal forces calculating.

(a) for shear force (b) for bending moment

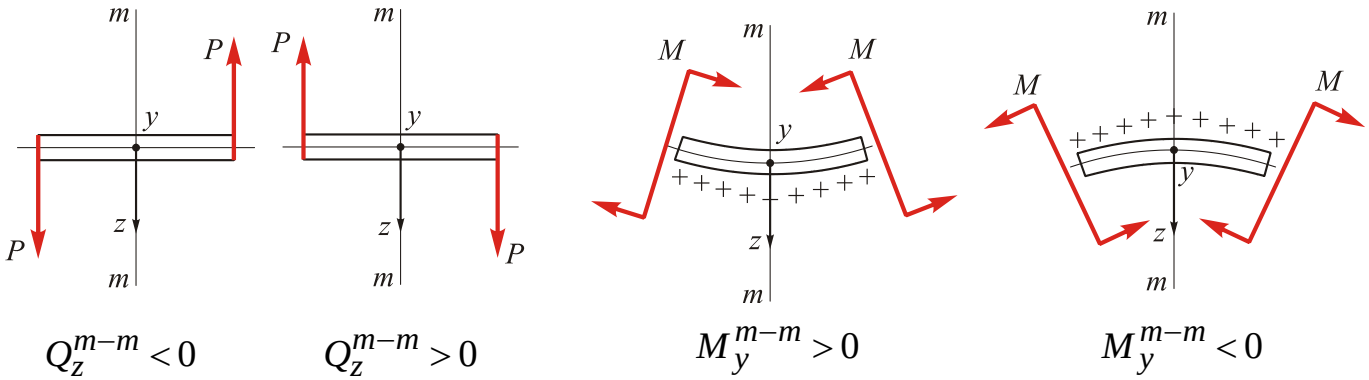



Fig. 1

2. Writing the equations of shear force and bending moment functions in an arbitrary sections of each potion (see Fig. 2).

Note. Method of sections will be applied from the right free tip of the cantilever and the cross-sections will be established at *x*-distance relative to origin of each potion.

I – I $(0 < x < a)$:

$$Q_z^I(x) = -qx\Big|_{x=0} = 0\Big|_{x=2} = -20 \text{ kN},$$

$$M_y^I(x) = -M + qx\frac{x}{2}\Big|_{x=0} = -20\Big|_{x=2} = 0 \text{ kNm}.$$

II – II $(0 < x < b)$:

$$Q_z^{II}(x) = -qa\Big|_{x=0} = -20\Big|_{x=2} = -20 \text{ kN},$$

$$M_y^{II}(x) = -M + qa\left(\frac{a}{2} + x\right)\Big|_{x=0} = 0\Big|_{x=2} = 40 \text{ kNm}.$$

III – III $(0 < x < c)$:

$$Q_z^{III}(x) = -qa + P - qx\Big|_{x=0} = 10\Big|_{x=2} = -10 \text{ kN},$$

Note, that the change of shear force sign within the boundaries of this section predicts the bending moment extreme value, since the derivative of bending moment is equal to shear force:

$$\frac{d(M_y^{III}(x))}{dx} = qa - P + qx = \left|Q_z^{III}(x)\right|.$$

Therefore, zero shear force and also zero bending moment derivative represent the point of bending moment extreme value.

To find it, let us determine the coordinate of zero shear force $x_e$ and substitute it into bending moment equation.

$$Q_z^{III}(x_e) = -qa + P - qx_e = 0 \rightarrow x_e = \frac{1}{q}(P - qa) = \frac{30 - 10 \times 2}{10} = 1 \text{ m}.$$

$$M_y^{III}(x) = -M + qa\left(\frac{a}{2} + b + x\right) - Px + qx\frac{x}{2}\Big|_{x=0} = 40\Big|_{x=2} = 40 \text{ kNm}.$$

$$M_y^{III}(x_e) = M_{y_{\max}}^{III} = -M + qa\left(\frac{a}{2} + b + x_e\right) - Px_e + \frac{qx_e^2}{2} = 35 \text{ kNm}.$$

3. Designing the graphs of shear forces and bending moments. Positive shear forces will be drawn upwards and vice versa. Bending moment graph will be drawn on tensile fibers according to the sign convention mentioned above (see Fig. 1 right). The graphs are shown on Fig. 2.

4. Applying the shear force and bending moment values in the rigid support as corresponding reactions according to accepted sign conventions (see Fig. 2).

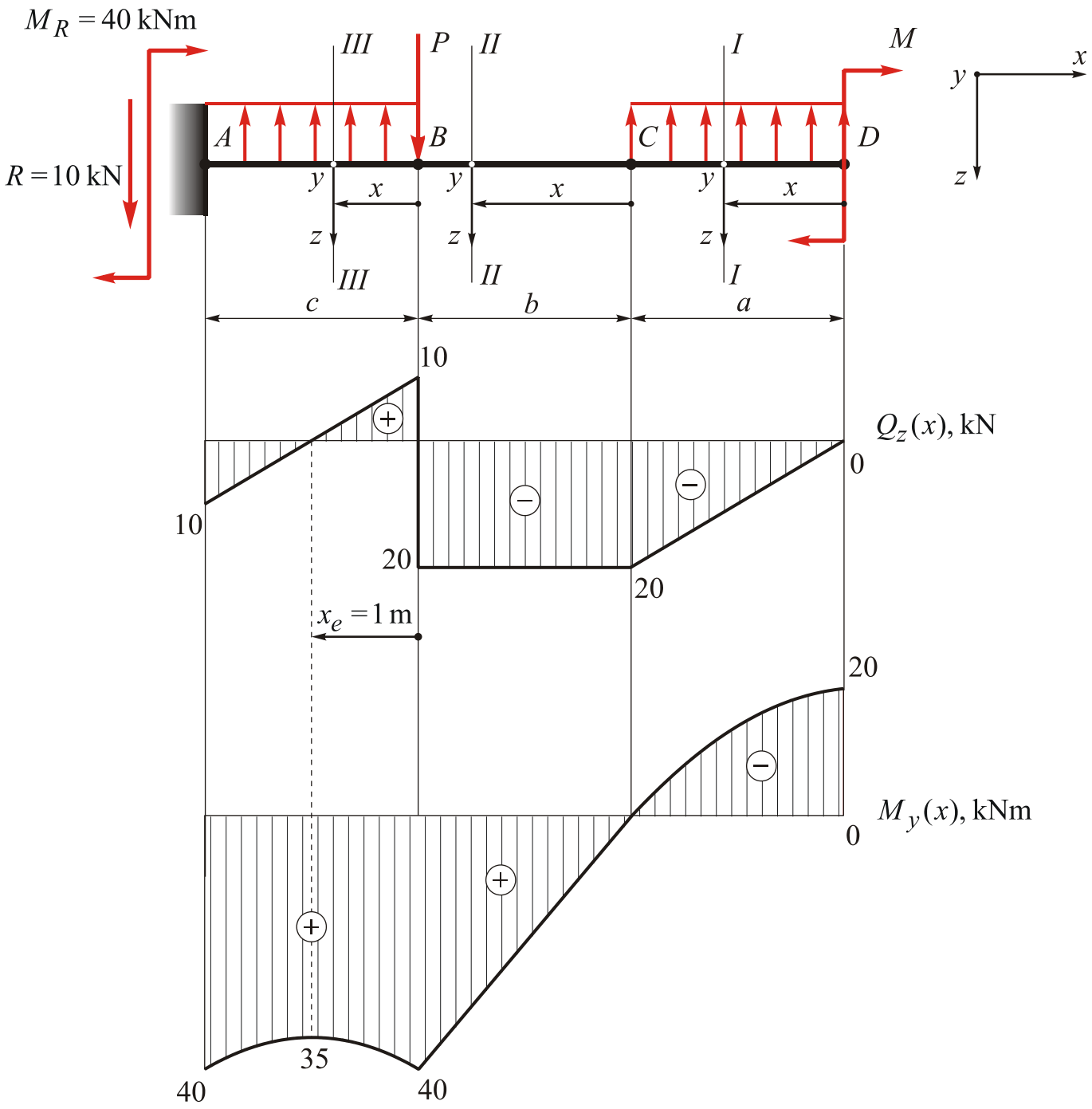

$M_R = 40$ kNm
$R = 10$ kN
III
P
II
I
M
A
B
C
D
x
y
z
y
x
z
III
II
I
c
b
a
10
$Q_z(x)$, kN
0
10
20
20
$x_e = 1$ m
20
$M_y(x)$, kNm
0
35
40
40

Fig. 2