MINISTRY OF EDUCATION AND SCIENCE OF UKRAINE

National aerospace university “Kharkiv Aviation Institute”

Department of aircraft strength

Course
Mechanics of materials and structures

HOME PROBLEM 5

Graphs of Shear Force and Bending Moment Distribution in Plane Bending (Two-Supported Beams)

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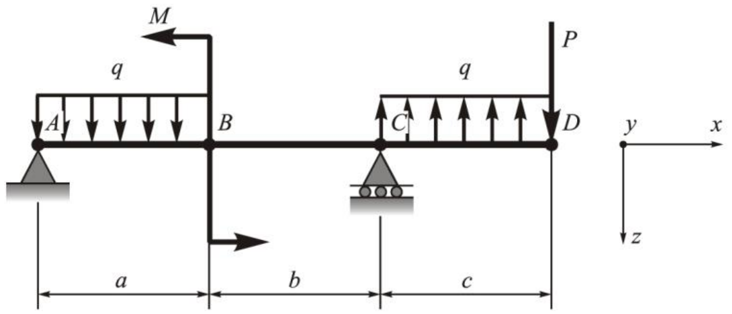
Subject: mechanics of materials
Document: home problem
Topic: graphs of shear force and bending moment distribution along the length of a beam in plane bending deformation.
Full name of the student, group

Variant: 1 **Complexity:** 1



Given: q = 10 kN/m, M = 20 kNm, P = 30 kN, a = 2 m, b = 4 m, c = 2 m.

Goal: obtain the equations of shear force and bending moment in the cross-sections of a beam and design the graphs of their distribution along the beam length.

Full name of the lecturer **signature**

Mark:

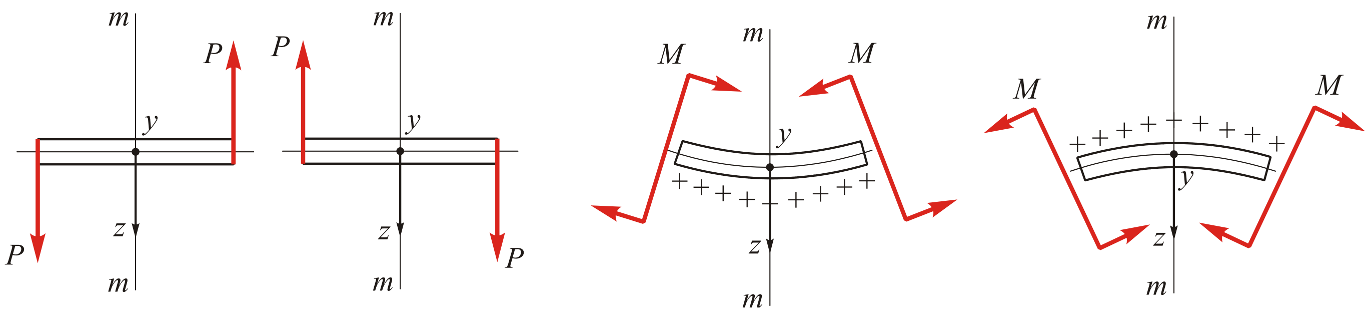
Shear force in a prismatic rod is equal to the algebraic sum of all external forces projections on the cross-section lying on one side of the section (left or right).

The bending moment at a section is equal to the sum of moments, in relevance to the transverse axis in the section, of all external forces applied to one side of the section (left or right).

Solution

1. Accepting the sign conventions in internal forces calculating.

(a) for shear force (b) for bending moment



$Q_z^{m-m} < 0$ $Q_z^{m-m} > 0$ $M_y^{m-m} > 0$ $M_y^{m-m} < 0$

Fig. 1

2. Calculating the reactions in supports R_A and R_C (see Fig. 2). Since their actual directions are unknown we will direct the reactions arbitrary, for example, upwards. The reaction positive sign from future calculating will mean that the reaction original direction is coincident with actual one and vice versa. For the reactions calculating we will use two momentum equations of equilibrium relative to supports (C and A points). Third equation of equilibrium in vertical direction we will use to check the result accuracy.

Note, that in designing the momentum balance equations clockwise rotation will be assumed to be positive and vice versa (see Fig. 2).

$$\sum M_A = 0 = +\frac{qa^2}{2} - M - R_C(a+b) - qa(\frac{a}{2}+b+c) + P(a+b+c),$$

$$R_C = \frac{1}{a+b}\left(-\frac{qa^2}{2} + M + qa(\frac{a}{2}+b+c) - P(a+b+c) \right) = +16,67\ \text{kN}.$$

$$\sum M_C = 0 = -\frac{qc^2}{2} - M + R_A(a+b) - qa(\frac{a}{2}+b) + Pc,$$

$$R_A = \frac{1}{a+b}\left(+\frac{qc^2}{2} + M + qa(\frac{a}{2}+b) - Pc \right) = +13.33\ \text{kN}.$$

$$\sum P_z = 0 = -R_A - R_C - qc + qa + P = -13.33 - 16.67 - 10\times 2 + 10\times 2 + 30 = 0.$$

3. Selecting the arbitrary cross-sections at x-distances from the origin of each potion and writing the equations of shear force and bending moment functions.

In this solution, we will consider the equilibrium of two left-situated parts of the rod (movement from left to right for portions I-I and II-II) and one right-situated part (movement from right to left for portion III-III). This is shown on Fig. 2. Note, that in such selection, the equations of internal forces will be the most simple in shape.

$\mathrm{I-I} \quad 0 < x < a:$

$$Q_z^I(x) = R_A - qx\,|_{x=0} = 13.33\,|_{x=2} = 13.33 - 20 = -6.67\ \text{kN},$$

$$M_y^I(x) = R_A x - \frac{qx^2}{2}\,|_{x=0} = 0\,|_{x=2} = 26.66 - 20 = +6.66\ \text{kNm}.$$

Note, that the change of shear force sign within the boundaries of this section predicts the bending moment extreme value, since the derivative of bending moment is equal to shear force:

$$\frac{d(M_y^I(x))}{dx} = R_A - qx = \left| Q_z^I(x) \right|.$$

Therefore, zero shear force and also zero bending moment derivative represent the point of bending moment extreme value.

To find it, let us determine the coordinate of zero shear force x_e and substitute it into bending moment equation.

$$Q_z^I(x_e) = 0 = R_A - qx_e = 13.33 - 10x_e, \quad x_e = 1.33\,\text{m}.$$

$$M_{y_{\max}}^I = M_y^I(x_e) = R_A x_e - \frac{qx_e^2}{2} = 13.33\times 1.33 - \frac{10}{2}\times 1.33^2 = +8.89\ \text{kNm}.$$

$\mathrm{II-II} \quad 0 < x < b:$

$$Q_z^{II}(x) = R_A - qa = 13.33 - 20 = -6,67\ \text{kN},$$

$$M_y^{II}(x) = R_A(a+x) - qa\left(\frac{a}{2}+x\right) - M\,|_{x=0} = 26.66 - 20 - 20 =$$

$$= -13.34\,|_{x=4} = 79.98 - 100 - 20 = -40\ \text{kNm}.$$

III – III $\quad 0 < x < c$:

$$Q_z^{III}(x) = P - qx\,|_{x=0} = 30\,|_{x=2} = 30 - 20 = 10 \text{ kN},$$

$$M_y^{III}(x) = -Px + \frac{qx^2}{2}\,|_{x=0} = 0\,|_{x=2} = -60 + 20 = -40 \text{ kNm}.$$

4. Designing the graphs of shear forces and bending moment distribution. Positive shear forces will be drawn upwards and vice versa. Bending moment graph will be drawn on tensile fibers according to the sign convention mentioned above (see Fig. 1). The graphs are shown on Fig. 2.

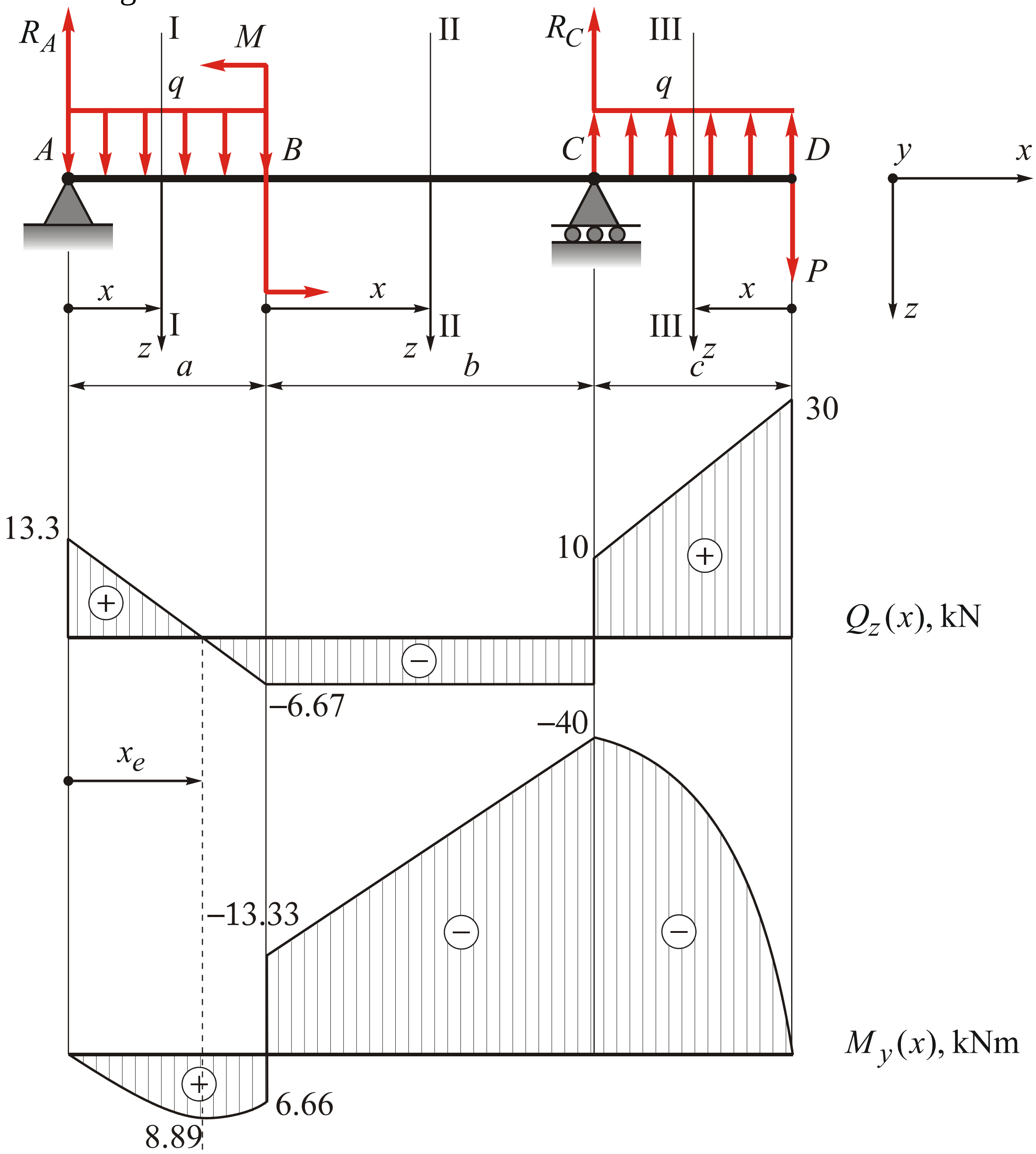


Fig. 2

5. Checking the results.

The “abrupts” on the Q_z graph are numerically equal to the force and reaction values in the points of these forces application.

The “abrupts” on the M_y graph are numerically equal to the concentrated moment values in the points of these moments application.