MINISTRY OF EDUCATION AND SCIENCE OF UKRAINE

National aerospace university “Kharkiv Aviation Institute”

Department of aircraft strength

Course
Mechanics of materials and structures

HOME PROBLEM 6

Graphs of Shear and Normal Forces and Bending Moment Distribution in Plane Bending of Statically Determinate Frames

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National aerospace university
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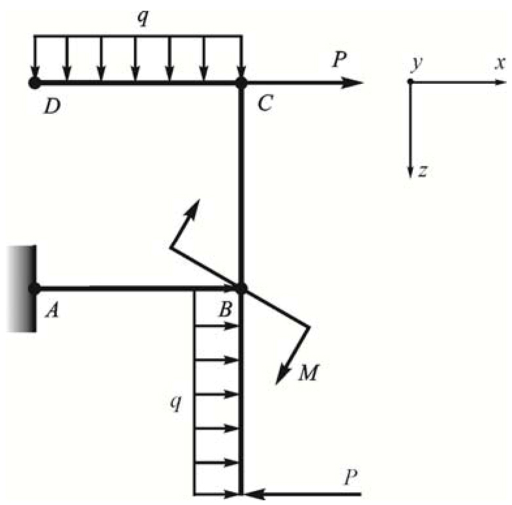
Subject: mechanics of materials
Document: home problem
Topic: graphs of shear force and bending moment distribution along the length of a beam in plane bending deformation.
Full name of the student, group

Variant: **1** **Complexity: 1**



Given: q = 10 kN/m, M = 30 kNm, P = 40 kN, l = 2 m.

Goal: obtain the equations of shear and normal forces and also bending moment in the cross-sections of a frame and design the graphs of their distribution along the frame portion length.

Full name of the lecturer **signature**

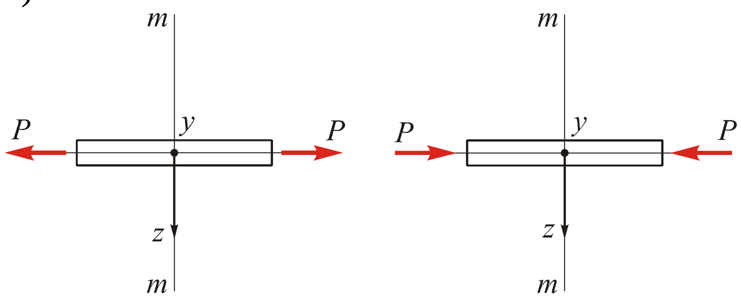
Mark:

In normal force calculating, we will use the rule that normal force in the cross-section is numerically equal to algebraic sum of external forces applied to the right or to the left part of the rod after its virtual cutting according to the method of sections. Tensile external force should be substituted into the equation with positive sign and visa versa. This sign convention is shown on Fig. 1.

Shear force in an arbitrary section is equal to the algebraic sum of all external forces projections on the z-axis of cross-section, but lying only on one side of the section (left or right) (see Fig. 1).

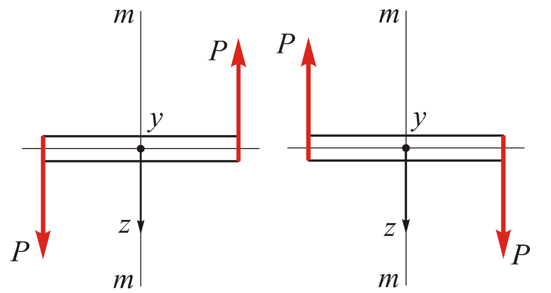
The bending moment in a section is equal to the sum of moments, in relevance to the transverse axis in the section, of all external forces applied to one side of the section (left or right) (see Fig. 1).

a) for normal force



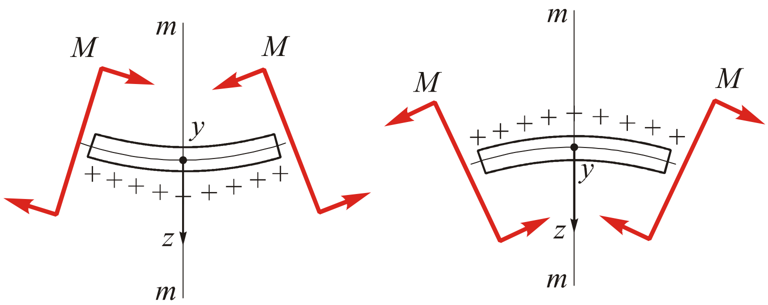
$N_x^{m-m} > 0$ $N_x^{m-m} < 0$

b) for shear force



$Q_z^{m-m} < 0$ $Q_z^{m-m} > 0$

б) for bending moments



$M_y^{m-m} > 0$ $M_y^{m-m} < 0$

Fig. 1

Comment: in the case when the curvature of deflected beam curve is coincident with z-axis direction, corresponding component of bending moment equation will be assumed to be positive and vice versa. The graph of bending moment will be designed on tensile fibers of the beam since position of tensile fiber is clear in both situations shown on Fig. 1.

Solution

1. Drawing the frame in scale and applying the support reactions in arbitrary directions.

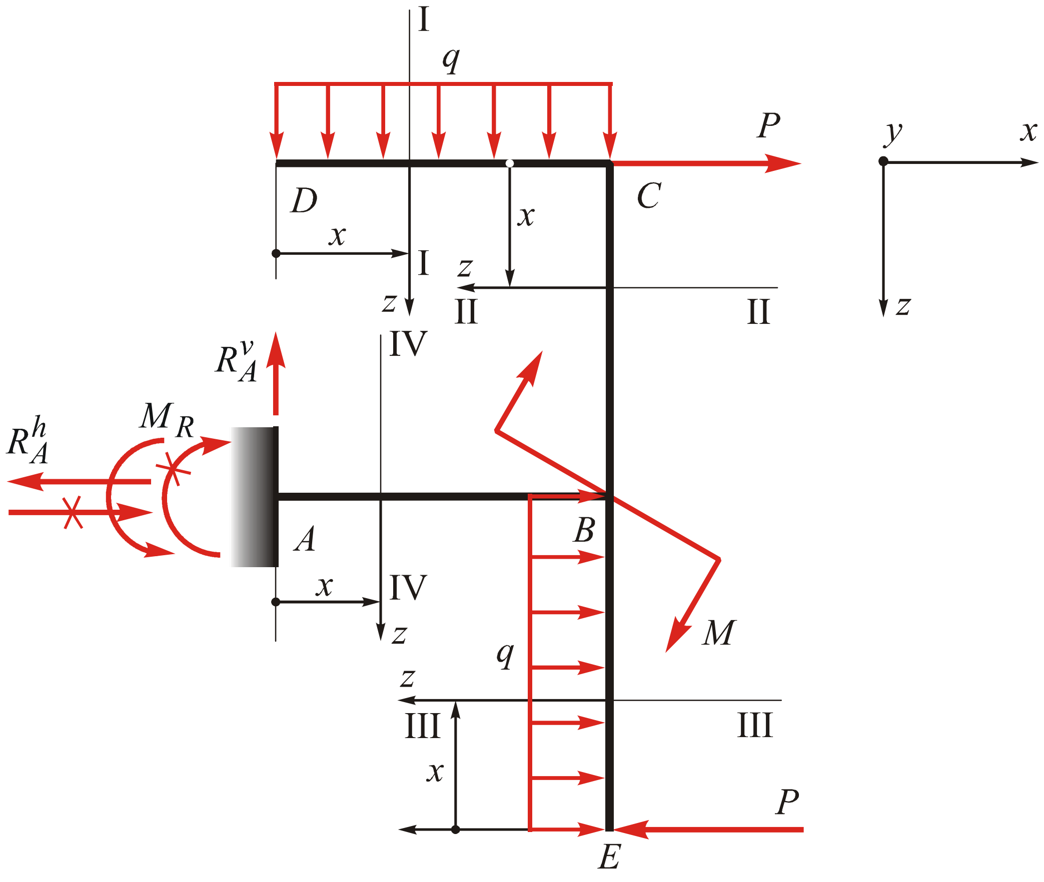


Fig. 2

2. Calculating the reactions in supports R_A^h, R_A^v, M_R.

Since the reactions actual directions are unknown we will direct the reactions arbitrary (see Fig. 2). The reaction positive sign from future calculating will mean that the reaction original direction is coincident with actual one and vice versa. In reactions calculating, we will use two momentum equations of equilibrium (relative to *A* and *C* points) and also equation of force equilibrium in vertical direction.

Note, that in designing the momentum equations of equilibrium clockwise rotation will be assumed to be negative and vice versa.

(1) $\sum M_A = -ql\dfrac{l}{2} - M - Pl - Pl + ql\dfrac{l}{2} - M_R = 0,$

$$M_R = -q\frac{l^2}{2} - M - 2Pl + \frac{ql^2}{2} = 30 - 80 - 80 = -190 \text{ kNm}.$$

"Minus" sign of M_R moment illustrates that its actual direction is opposite to preliminary assumed i.e. M_R is directed counterclockwise. It is shown on Fig. 2.

(2) $\sum M_D = -ql\dfrac{l}{2} + R_A^h l - M + ql\left(l + \dfrac{l}{2}\right) - 2Pl + M_R = 0,$

$$R_A^h = \frac{\dfrac{ql^2}{2} + M - ql\left(l + \dfrac{l}{2}\right) + 2Pl - M_R}{l} = \frac{20 + 30 - 60 + 160 - 190}{2} = -20 \text{ kN}.$$

"Minus" sign of R_A^h reaction illustrates that its actual direction is opposite to preliminary assumed i.e. R_A^h is directed to left. This is shown on Fig. 2.

(3) $\sum P_z = ql - R_A^v = 0$, $R_A^v = ql = 10 \times 2 = +20$ kN.

3. Selecting the arbitrary cross-sections at *x*-distances from the origin of each potion and writing the equations of normal and shear forces and also bending moment functions.
In this solution, the portion balance will be considered to get the most simple equations of internal forces: the portions *I-I* and *II-II* will be considered from *D* point (motion from *D* to *B* point), potion *III-III* will be considered upward from *E* point and last portion will be considered from *A* point to right. This is shown on Fig. 2.

$I - I \quad (0 < x < l)$

$N_x^I(x) = 0$ kN,

$Q_z^I(x) = -qx|_{x=0} = 0|_{x=2} = -20$ kN,

$M_z^I(x) = -\dfrac{qx^2}{2}|_{x=0} = 0|_{x=2} = -20$ kNm.

$II - II \quad (0 < x < l)$

$N_x^{II}(x) = -ql = -20$ kN,

$Q_z^{II}(x) = +P = 40$ kN,

$M_y^{II}(x) = -\dfrac{ql^2}{2} + Px|_{x=0} = -20|_{x=2} = 60$ kNm.

$III - III \quad (0 < x < l)$

$N_x^{III}(x) = 0$ kN,

$Q_z^{III}(x) = +P - qx|_{x=0} = 40|_{x=2} = 20$ kN,

$M_y^{III}(x) = -Px + \dfrac{qx^2}{2}|_{x=0} = 0|_{x=2} = -80 + 20 = -60$ kNm.

$IV - IV, \quad (0 < x < l)$

$N_x^{IV}(x) = +R_A^h = 20$ kN,

$Q_z^{IV}(x) = +R_A^v = 20$ kN,

$M_y^{IV}(x) = R_A^v x - M_R|_{x=0} = -190|_{x=2} = 40 - 190 = -150$ kNm.

4. Designing the graphs of normal and shear forces and also bending moment distribution. Bending moment graph will be drawn on tensile fibers according to the sign convention mentioned above (see Fig. 1). The graphs are shown on Fig. 3.

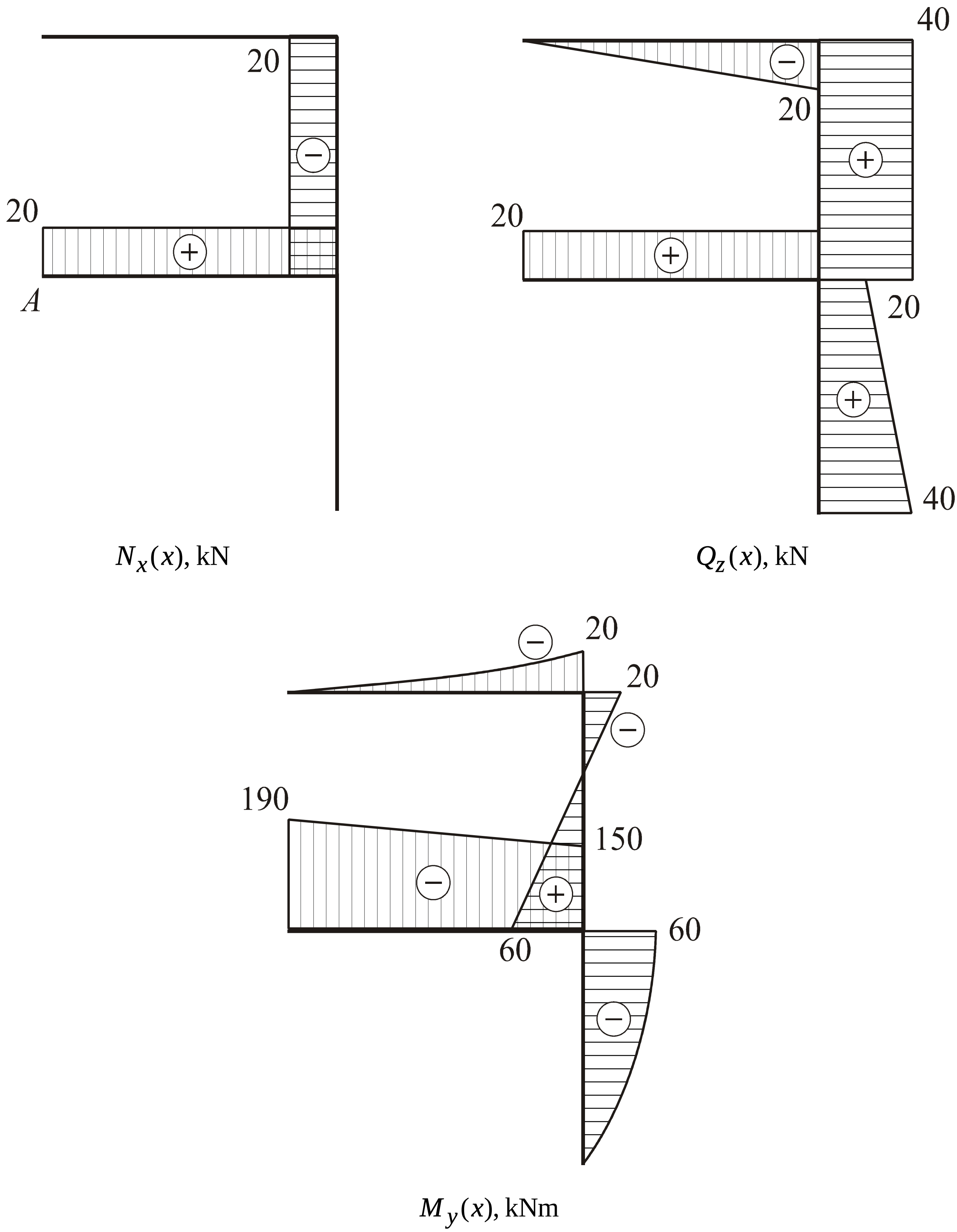


Fig. 3

5. Checking the balance of two infinitely little elements of the frame.

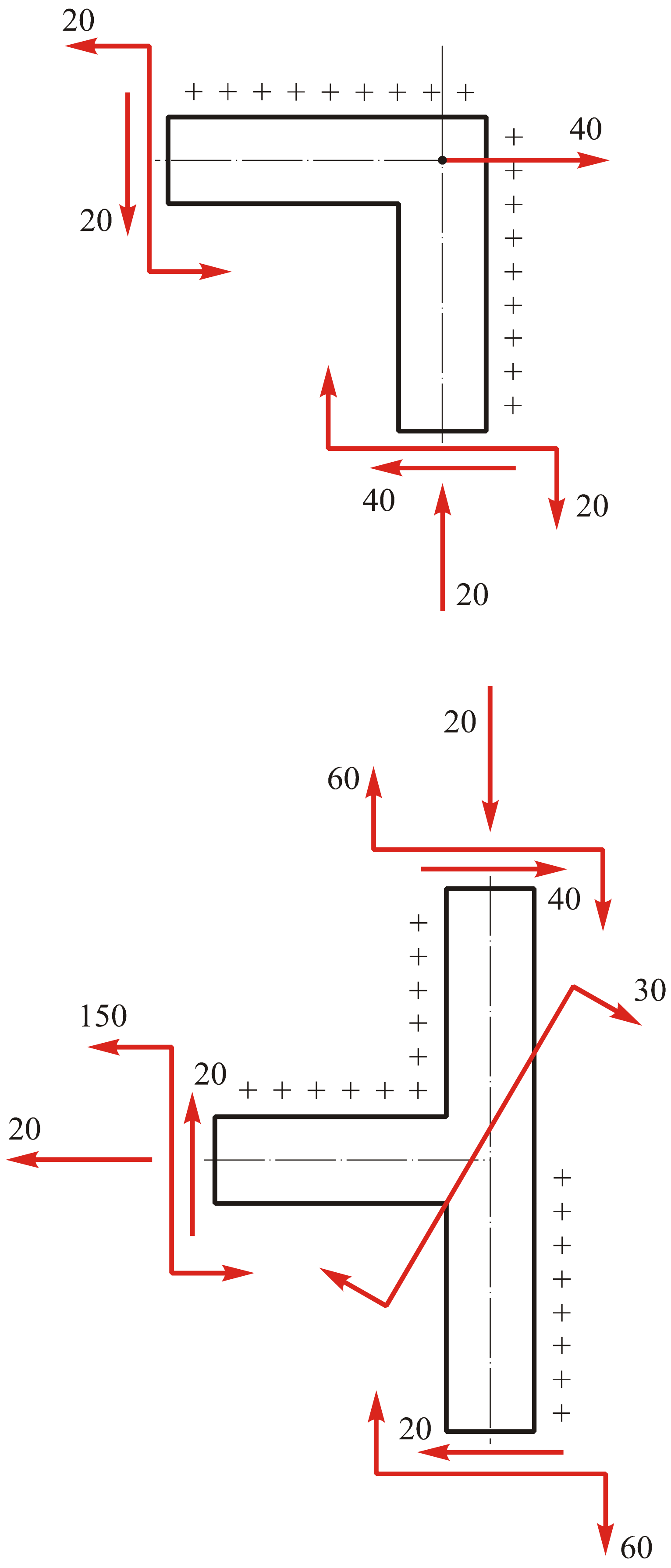


Fig. 4