MINISTRY OF EDUCATION AND SCIENCE OF UKRAINE

National aerospace university “Kharkiv Aviation Institute”

Department of aircraft strength

Course
Mechanics of materials and structures

HOME PROBLEM 7

Design problem for statically indeterminate rod in tension-compression

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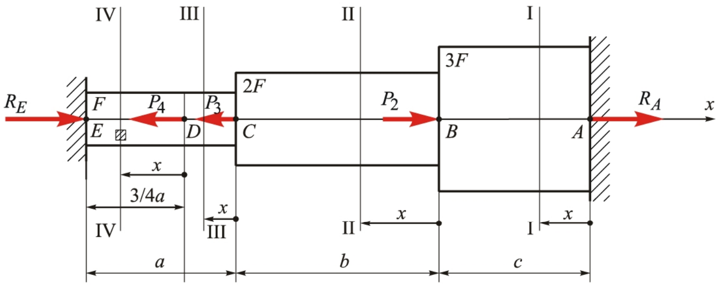
National aerospace university
“Kharkiv Aviation Institute”
Department of aircraft strength

Subject: mechanics of materials
Document: home problem
Topic: Stresses and elongations in statically indeterminate rods in tension-compression
Full name of the student, group

Variant: 1 **Complexity:** 1



Given: $[\sigma]_t = 160$ MPa; $[\sigma]_c = 200$ MPa;
$P_2 = 10$ kN; $P_3 = 50$ kN, $P_4 = 80$ kN;
$a = 3$ m, $b = 4$ m, $c = 5$ m.

Goal:
1) open static indeterminacy and design the graph on normal forces;
2) calculate cross-sectional area F;
3) calculate acting stresses in the portions of the rod and design the graph of their distribution along the length of the rod;
4) design the graph of the rod elongations;
5) estimate stress state in critical cross-section.

Full name of the lecturer **signature**

Mark: **Mark:**

General method in statically indeterminate rods and rod system analysis is in finding complementary equations of deformation compatibility to determine internal forces in the rod. The number of compatibility equations depends on degree of static indeterminacy.

In our case, degree of static indeterminacy $k = m - n$, where $m = 2$ – total number of constraints (reactions), $n = 1$ – number of equations of equilibrium.

After substituting

$k = 2 - 1 = 1$.

Conclusion: the rod is singly statically indeterminate.

Due to axial loading, only axial reactions R_A and R_E take place in this problem.

Solution

1. Calculating the support reactions R_A and R_E (see Fig. 1).

(a) from condition of equilibrium $\sum F_x = 0$. Direction to the right is assumed to be positive (see Fig. 1).

$$\sum F_x = 0 = R_E - P_4 - P_3 + P_2 + R_A = 0.$$

(b) designing the compatibility equation.

It is evident that this deformable rod has two immobile cross-sections A and E. Therefore total elongation of the rod is zero, i.e. $\Delta l_{AE} \equiv 0$:

$$\Delta l_{AE} = \Delta l_{AB} + \Delta l_{BC} + \Delta l_{CD} + \Delta l_{DE}.$$

The elongations of particular portions AB, BC, CD, DE are generated by corresponding normal forces. According to the method of sections the equations of normal forces are the following:

I–I $(0 < x < c)$

$$N_x^I(x) = +R_A.$$

II–II $(0 < x < b)$

$$N_x^{II}(x) = +R_A + P_2.$$

III–III $(0 < x < a/4)$

$$N_x^{III}(x) = +R_A + P_2 - P_3.$$

IV–IV $(0 < x < 3a/4)$

$$N_x^{IV}(x) = +R_A + P_2 - P_3 - P_4.$$

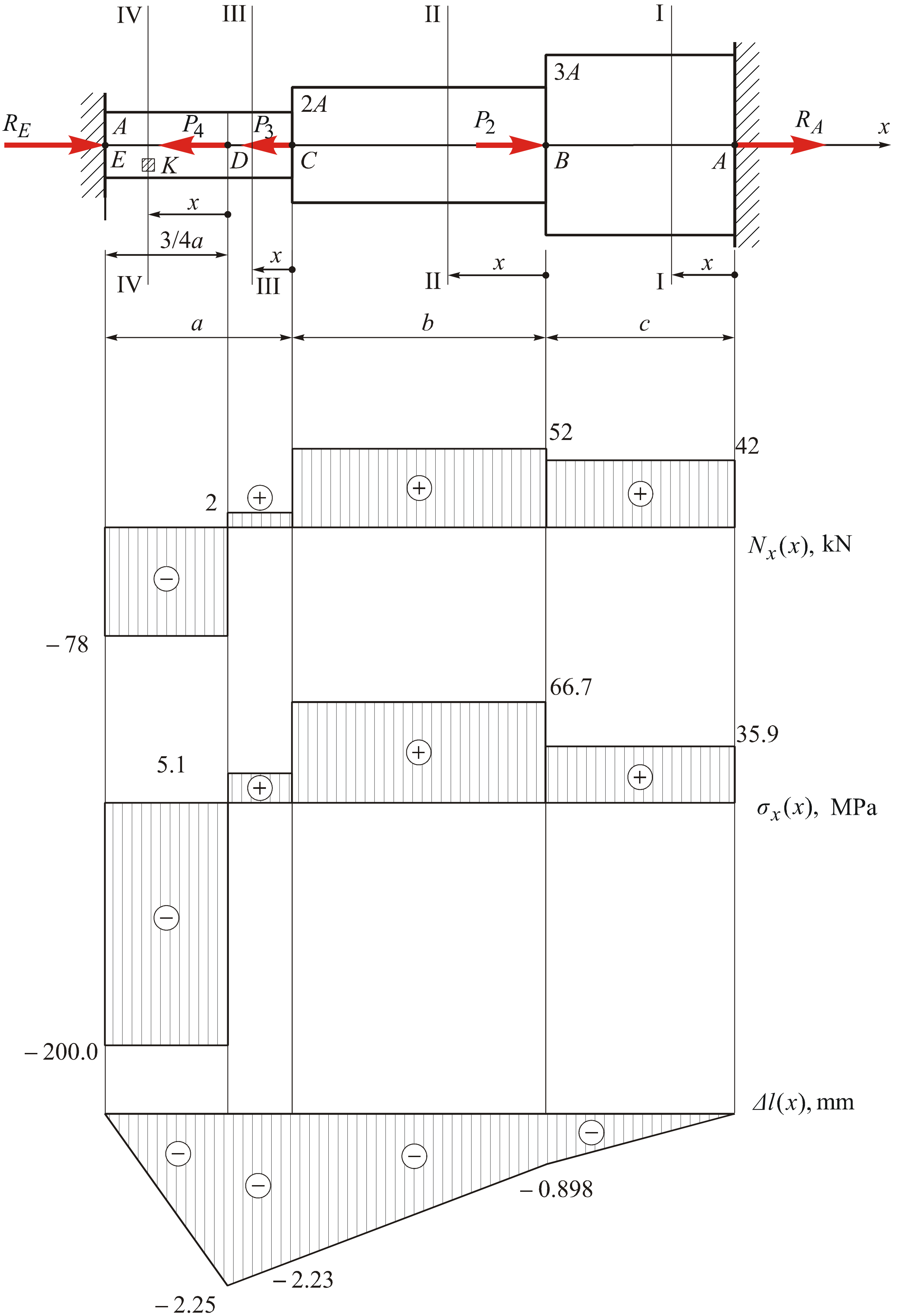

IV
III
II
I
R_E
A
P_4
P_3
$2A$
P_2
$3A$
R_A
x
E
K
D
C
B
A
x
$3/4a$
x
x
x
IV
III
II
I
a
b
c
52
42
2
$N_x(x)$, kN
– 78
66.7
35.9
5.1
$\sigma_x(x)$, MPa
– 200.0
$\Delta l(x)$, mm
– 0.898
– 2.23
– 2.25

Fig. 1

Corresponding elongations of the portions are:

$$\Delta l_{AB}=\frac{N_x^I(x)(c)}{3AE};\quad \Delta l_{BC}=\frac{N_x^{II}(x)(b)}{2AE};\quad \Delta l_{CD}=\frac{N_x^{III}(x)\left(\frac{1}{4}a\right)}{AE};$$

$$\Delta l_{DE}=\frac{N_x^{IV}(x)\left(\frac{3}{4}a\right)}{AE};$$

$$\rightarrow\frac{5}{3}R_A+2(R_A+P_2)+\frac{3}{4}(R_A+P_2-P_3)+\frac{9}{4}(R_A+P_2-P_3-P_4)=0\rightarrow$$

$$\rightarrow 80R_A=-60P_2+36P_3+27P_4=-600+1800+2160=3360\rightarrow R_A=\frac{3360}{80}=+42\ \text{kN}.$$

"+" sign of R_A reaction supports the conclusion that R_A reaction is actually determined to the right. After R_A finding, static indeterminacy is opened and normal forces may be determined.

2. Calculating the normal forces in an arbitrary cross-section of each portion.

$$N_x^I(x)=R_A=+42\ \text{kN},$$
$$N_x^{II}(x)=+R_A+P_2=+42+10=52\ \text{kN},$$
$$N_x^{III}(x)=R_A+P_2-P_3=+42+10-50=+2\ \text{kN},$$
$$N_x^{IV}(x)=R_A+P_2-P_3-P_4=+42+10-50-80=-78\ \text{kN}.$$

The graph of normal force distribution is shown on Fig. 1.

3. Calculating the cross-sectional area A from condition of strength in critical portion.

Due to $[s]_t\neq[s]_c$, it will be necessary to design two conditions of strength – for critically tensile and critically compressed portions. In our case,

(a) for three tensile portions *II-II* portion is evidently critical after comparing the relations $\frac{42}{3A}$, $\frac{52}{2A}$ and $\frac{2}{A}$. That is why

$$\sigma_{\max t}=\sigma_x^{II}=\frac{N_x^{II}(x)}{2A_t}\leq[\sigma]_t\rightarrow A_t=\frac{N_x^{II}(x)}{2[\sigma]_t}=\frac{52\times10^3}{2\times160\times10^6}=1.625\times10^{-4}\ \text{m}^2;$$

(b) for compressed portion:

$$\left|\sigma_{\max c}\right|=\left|\sigma_x^{IV}\right|=\frac{\left|N_x^{IV}(x)\right|}{A_c}\leq[\sigma]_c\rightarrow A_c=\frac{N_x^{IV}(x)}{[\sigma]_c}=\frac{78\times10^3}{200\times10^6}=3.9\times10^{-4}\ \text{m}^2.$$

For future calculating, we should select larger of two cross-sectional areas which will satisfy both conditions of strength:

$$A_{\max}=A_c=3.9\times10^{-4}\ \text{m}^2.$$

4. Calculating the acting stresses.

$$\sigma_x^I=\frac{N_x^I(x)}{3A_{\max}}=\frac{42\times10^3}{3\times3.9\times10^{-4}}=+35.9\ \text{MPa},$$

$$\sigma_x^{II}=\frac{N_x^{II}(x)}{2A_{\max}}=\frac{52\times10^3}{2\times3.9\times10^{-4}}=+66.7\ \text{MPa},$$

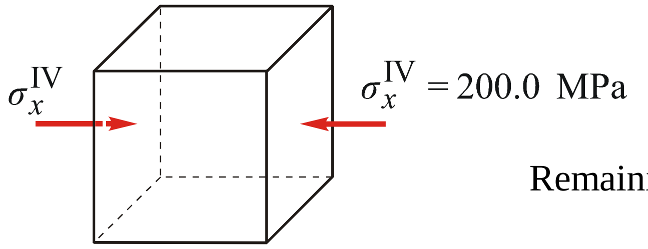
$$\sigma_x^{III} = \frac{N_x^{III}(x)}{A_{\max}} = \frac{+2\times10^3}{3.9\times10^{-4}} = +5.1 \text{ MPa},$$

$$\sigma_x^{IV} = \frac{N_x^{IV}(x)}{A_{\max}} = \frac{-78\times10^3}{3.9\times10^{-4}} = -200.0 \text{ MPa}.$$

Graph of stress distribution is shown on Fig. 1.

5. Analysis of stress state type in an arbitrary point K of critical section.

point K



$s_x^{IV} = s_3 = -200$ MPa,

Remaining principal stresses are:

$s_1 = s_2 = 0.$

Fig. 2

Conclusion: stress state is uniaxial, deformation is tension.

6. Drawing the graph of displacements.

E point is selected as the origin. The displacements of particular points are designated by δ. Therefore, $\delta_E = 0$. Due to Hook's law, the displacement function is linear, that is why the displacements of each portion tip are numerically equal to the portion elongation or shortening. Elasticity modulus value $E = 2\times10^{11}$ Pa is used in this calculation.

$$\delta_D = \Delta l_{ED} = \frac{N_x^{IV}(x)3a}{4EA_{\max}} = \frac{-78\times10^3\times3\times3}{4\times2\times10^{11}\times3.9\times10^{-4}} = -22.5\times10^{-4} \text{ m} = -2.25 \text{ mm}.$$

$$\delta_C = \Delta l_{EC} = \Delta l_{ED} + \Delta l_{DC} = -22.5\times10^{-4} + \frac{N_x^{III}(x)a}{4EA_{\max}} = -22.5\times10^{-4} +$$

$$+\frac{2\times10^3\times3}{4\times2\times10^{11}\times3.9\times10^{-4}} = -22.5\times10^{-4} + 0.19\times10^{-4} = -22.31\times10^{-4} \text{ m} = -2.23 \text{ mm}.$$

$$\delta_B = \Delta l_{EB} = \Delta l_{EC} + \Delta l_{CB} = -22.31\times10^{-4} + \frac{N_x^{II}(x)b}{2EA_{\max}} = -22.31\times10^{-4} +$$

$$+\frac{52\times10^3\times4}{2\times2\times10^{11}\times3.9\times10^{-4}} = -22.31\times10^{-4} + 13.33\times10^{-4} = -8.98\times10^{-4} \text{ m} = -0.898 \text{ mm}.$$

$$\delta_A = \Delta l_{EA} = \Delta l_{EB} + \Delta l_{BA} = -8.98\times10^{-4} + \frac{N_x^{I}(x)c}{3EA_{\max}} = -8.98\times10^{-4} +$$

$$+\frac{42\times10^3\times5}{3\cdot2\times10^{11}\times3.9\times10^{-4}} = -8.98\times10^{-4} + 8.97\times10^{-4} = 0.01\times10^{-4} \text{ m}.$$

Let us estimate the error of calculating, since really the displacement of A point must be zero according to compatibility equation:

$$D = \frac{0.01\times10^{-4}}{8.97\times10^{-4}}\times100\% = 0.11\%.$$

Due to negligibly little error, the calculation is true.

The graph of displacements is also shown on Fig. 1.