MINISTRY OF EDUCATION AND SCIENCE OF UKRAINE

National aerospace university “Kharkiv Aviation Institute”

Department of aircraft strength

Course
Mechanics of materials and structures

HOME PROBLEM 8

Strength and Rigidity Analysis of Statically Determinate Shaft

Name of student:

Group:

Advisor:

Data of submission:

Mark:

National aerospace university
“Kharkiv Aviation Institute”
Department of aircraft strength

Subject: mechanics of materials
Document: home problem
Topic: Strength and Rigidity Analysis of Staticaly Determinate Shafts
Full name of the student, group

Variant: 1 **Complexity:** 1

Given: thickness ratio $\alpha = d/D = 0.8$; $[\tau] = 100$ MPa; $[\psi] = 1$ degree/m;
$G = 8 \cdot 10^{10}$ Pa; $M_1 = 10$ kNm; $M_2 = 70$ kNm; $M_3 = 40$ kNm;
$a = 2$ m; $b = 3$ m; $c = 4$ m.

Goal:
1) copy from home problem No3 graph of torsional moment distribution;
2) calculate the diameters of solid and hollow shafts using conditions of strength and rigidity;
3) draw the graphs of stress distributions in critical sections of solid and hollow shafts;
4) estimate the type of stress state in an arbitrary point of critical cross-section (selecting yourself solid or hollow shape of a shaft);
5) compare the weights of 1 meter-in-length solid and hollow strong shafts;
6) design the graph of twisting angle distribution for solid or hollow strong shaft (select youself).

Full name of the lecturer **signature**

Mark:

Design problem
for solid and hollow uniform shafts

Goal:
(1) to determine internal torque moment in the shaft cross-sections and to design the graph of it’s distribution (clockwise rotation is assumed to be positive);
(2) calculate the diameter of solid shaft satisfying conditions of strength and rigidity;
(3) calculate the diameters of hollow shaft satisfying conditions of strength and rigidity, taking into account thickness ratio $\alpha = d/D = 0.8$;
(4) draw the graphs of stress distribution in critical cross-sections of both shafts;
(5) determine the type of stress state in an arbitrary point in critical cross-section of solid shaft;
(6) compare the weights of one meter-in-length solid and hollow strong shafts;
(7) calculate the angles of rotation for solid shaft and design the graph of twisting angle distribution.

Solution

1. Determine internal torque moments in an arbitrary cross-sections of the shaft applying the method of sections. Corresponding sign conventions are shown on Fig. 1. The shaft is shown on Fig. 2.

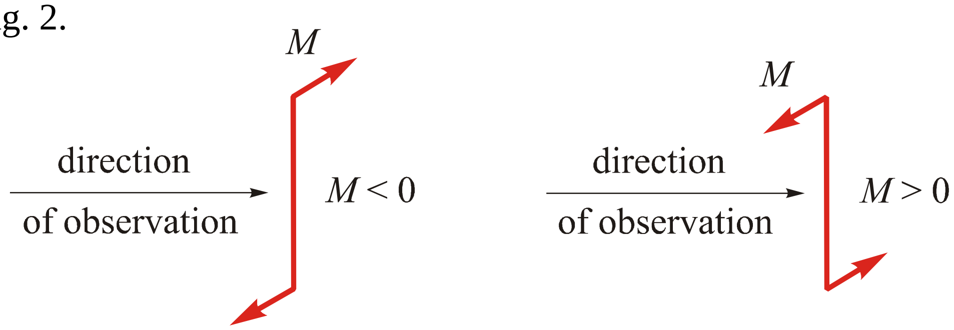

Fig. 1

I-I $(0 < x < c)$ $M_x^{I}(x) = M_3 = 40$ kNm,

II-II $(0 < x < b)$ $M_x^{II}(x) = M_3 - M_2 = 40 - 70 = -30$ kNm,

III-III $(0 < x < a)$ $M_x^{III}(x) = M_3 - M_2 + M_1 = 40 - 70 + 10 = -20$ kNm.

The graph $M_x(x)$ is shown on Fig. 2.

In result, $|M_x(x)| = 40$ kNm and I–I section is critical.

Note, that the graph $M_x(x)$ demonstrates that $M_x^{III}(x) = -20\text{ kNm}$. It allows determining the value of unknown moment M_0. Actually it has opposite direction of rotation, which is shown on Fig. 2 according to Fig. 1.

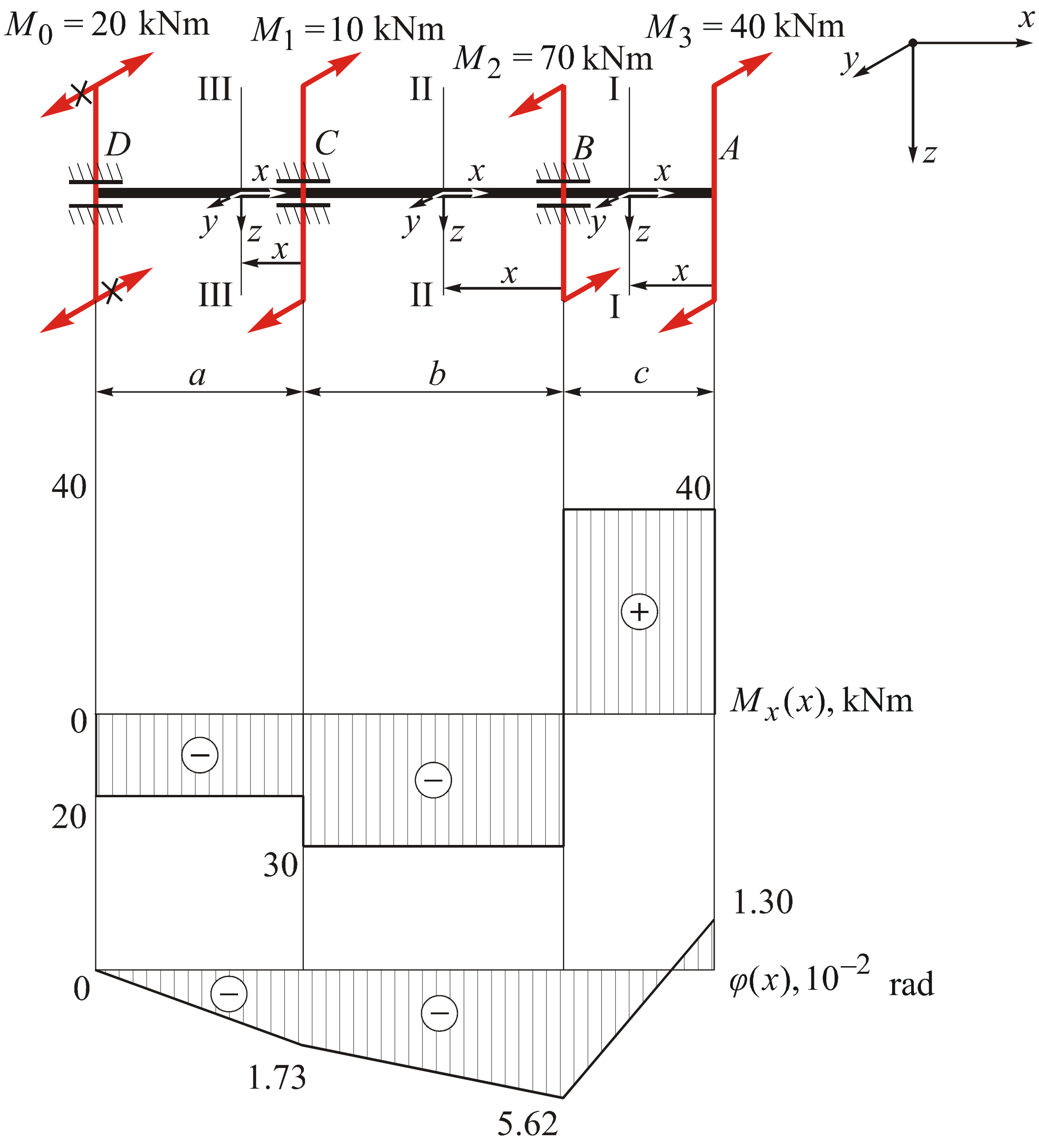

Fig. 2

2. Calculating the diameter of solid shaft:

2.1 satisfying the condition of strength:

$$\tau_{max}^{\bullet} = \frac{\left|M_{x_{max}}\right|}{W_\rho^{\bullet}} \le [\tau] \to D^{\bullet} \ge \sqrt[3]{\frac{16\left|M_{x_{max}}\right|}{\pi[\tau]}} = \sqrt[3]{\frac{16\times 40\times 10^3}{3{,}14\times 100\times 10^6}} = 0{,}127 \text{ m.}$$

2.2 satisfying the condition of rigidity:

$$\psi_{\max} = \frac{\left|M_{x_{\max}}\right|}{GI_{\rho}} \le [\psi] \to D^{\oslash} \ge \sqrt[4]{\frac{32\left|M_{x_{\max}}\right|}{\pi G[\psi]} \cdot \frac{180}{\pi}} = \sqrt[4]{\frac{32\times 40\times 10^{3}\times 180}{3{,}14^{2}\times 8\times 10^{10}\times 1}} = 0{,}131\ \text{m}.$$

Finally, the solid shaft diameter is $D^{\oslash} = 0{,}131$ **m. Note, it was calculated satisfying the condition of rigidity, i.e.** $y_{\max} = [y]$**.**

3. Calculating the diameters of hollow shaft in $\alpha = d/D = 0{,}8$:

3.1 satisfying the condition of strength:

$$\tau^{\circledcirc} = \frac{\left|M_{x_{\max}}\right|}{W_{\rho}^{\circledcirc}} \le [\tau],$$

$$W_{\rho}^{\circledcirc} = \frac{\pi D^{\circledcirc 3}}{16}\left(1-\alpha^{4}\right) \to D^{\circledcirc} = \sqrt[3]{\frac{16\left|M_{max}\right|}{\pi[\tau](1-\alpha^{4})}} = \sqrt[3]{\frac{16\times 40\times 10^{3}}{3{,}14\times 100\times 10^{6}\times 0{,}5904}} = 0{,}151\ \text{m},$$

$$d^{\circledcirc} = 0{,}8D^{\circledcirc} = 0{,}121\ \text{m}.$$

3.2 satisfying the condition of rigidity:

$$\psi_{\max} = \frac{\left|M_{max}\right|}{GI_{\rho}^{\circledcirc}} \le [\psi] \to I_{\rho}^{\circledcirc} = \frac{\pi D^{4}}{32}\left(1-\alpha^{4}\right) \to D^{\circledcirc} = \sqrt[4]{\frac{32\left|M_{max}\right|}{\pi G[\psi](1-\alpha^{4})} \times \frac{180}{\pi}} =$$

$$= \sqrt[4]{\frac{32\times 50\times 10^{3}\times 180}{3{,}14^{2}\times 8\times 10^{10}\times 1\times 0{,}5904}} = 0{,}149\ \text{m}, \quad d^{\circledcirc} = 0{,}8D^{\circledcirc} = 0{,}119\ \text{m}.$$

Finally, the hollow shaft diameters are $D^{\circledcirc} = 0{,}151$ **m and** $d^{\circledcirc} = 0{,}8D^{\circledcirc} = 0{,}121$ **м. Note, they were calculated satisfying the condition of strength, i.e.** $\tau_{\max} = [\tau]$**.**

4. Drawing the graphs of stress distribution in critical cross-section for two designed shafts.

4.1 for solid shaft:

$$\tau_{\max} = \frac{M_{x\max}}{W_{\rho}} = \frac{16M_{x\max}}{\pi D^{\oslash 3}} = \frac{16\times 40\times 10^{3}}{3.14\times(0.131)^{3}} = 90.7\ \text{MPa}.$$

Note, that $\tau_{\max}$ **is less than allowable stress for the shaft material since its diameter was calculated satisfying condition of rigidity, i.e.** $\psi_{\max} = [\psi]$**.**

4.2 for hollow shaft:

$$\tau_{\max} = \frac{M_{x\max}}{W_{\rho}} = \frac{16M_{x\max}}{\pi D^{\circledcirc 3}(1-\alpha^{4})} = \frac{16\times 40\times 10^{3}}{3.14\times(0.151)^{3}(1-0.8^{4})} = 100\ \text{MPa}.$$

Note, that $\tau_{\max}$ **is equal to allowable stress for the shaft material since its diameters were calculated satisfying condition of strength, i.e.** $\tau_{\max} = [\tau]$**.**

The graphs of stress distribution are shown on Fig. 3.

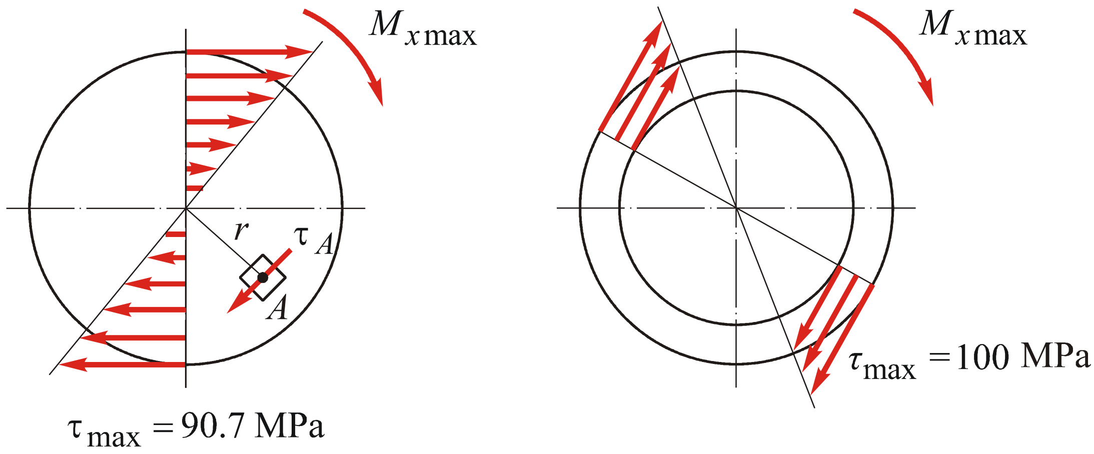

Fig. 3

5. Analysis of the stress state type in an arbitrary point A of critical section of solid shaft with polar coordinate $r = D/10$.

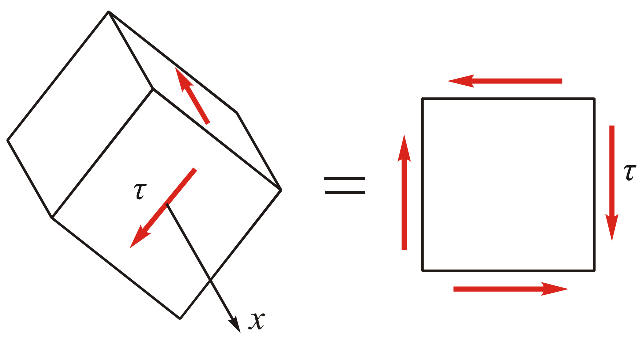

Fig. 3

$$t = \frac{M_{x\max}}{I_r} \times \frac{D}{10} = \frac{32 M_{x\max}}{p D^4} \times \frac{D}{10} = \frac{3.2 M_{x\max}}{p D^3} = \frac{3.2 \times 40 \times 10^3}{3.14 \times 0.131^3} = 18.1\,\text{MPa}.$$

Conclusion: stress state is biaxial and deformation is pure shear.

6. Comparing the weights of one meter-in-length segments of strong solid and hollow shafts.

The left graph of stress distribution shows that the core of solid shaft is under the action of low stresses, i.e. this part of cross-section is underloaded and can potentially withstand larger stresses. This part of the cross-section material may be used more effectively if it will be situated as far as possible from the cross-section centroid. Taking into consideration this idea, hollow shaft must be more effective in weight to withstand the same torque moment.

Check this idea and find relationship of the weights of the same in length solid and hollow shafts:

$$\frac{G^{\oslash}}{G^{\circledcirc}} = \frac{\gamma V^{\oslash}}{\gamma V^{\circledcirc}} = \frac{A^{\oslash} \times 1\,\text{m}}{A^{\circledcirc} \times 1\,\text{m}} = \frac{4\pi D^{\oslash 2}}{4\left(\pi D^{\circledcirc 2} - \pi d^{\circledcirc 2}\right)} = \frac{D^{\oslash 2}}{D^{\circledcirc 2} - d^{\circledcirc 2}}.$$

Note, that to compare the weights, we will use the diameters which were determined from conditions of strength. Then

$$\frac{G^{\oslash}}{G^{\circledcirc}} = \frac{0.127^2}{0.151^2 - 0.121^2} = 1.97\,.$$

Conclusion. Solid shaft is approximately 2 times greater in weight than hollow one due to more effective distribution of material in hollow shaft. This advantage will depend on the thickness ratio α: increase of α leads to decrease of hollow shaft weight.

7. Designing the graph of twisting angle distribution for solid shaft.
Note, that its diameter is $D^{\oslash} = 0.131$ m.
To design the graph, we will use the formula

$$\varphi(x) = \frac{M_x(x)}{GI_\rho} = kx$$ – linear function of the shaft length.

Also, angles of twist for *C, B, A* cross-sections we will calculate relative to reference *D*-section:

$$\varphi_C = \varphi_{DC} = \frac{M_x^{III} a}{GI_\rho} = \frac{-20\times10^3\times2\times32}{8\times10^{10}\times3.14\times0.131^4} = -1.73\times10^{-2}\ \text{rad.}$$

$$\varphi_B = \varphi_{DB} = \varphi_{DC} + \varphi_{CB} = \varphi_{DC} + \frac{M_x^{II} b}{GI_\rho} = -1.73\times10^{-2} + \frac{-30\times10^3\times3\times32}{8\times10^{10}\times3.14\times0.131^4} =$$

$$= -1.73\times10^{-2} - 3.89\times10^{-2} = -5.62\times10^{-2}\ \text{rad.}$$

$$\varphi_A = \varphi_{DA} = \varphi_{DB} + \varphi_{BA} = \varphi_{DB} + \frac{M_x^{I} c}{GI_\rho} = -5.62\times10^{-2} + \frac{40\times10^3\times4\times32}{8\times10^{10}\times3.14\times0.131^4} =$$

$$= -5.62\times10^{-2} + 6.92\times10^{-2} = +1.30\times10^{-2}\ \text{rad.}$$

Corresponding graph is shown on Fig. 2.