MINISTRY OF EDUCATION AND SCIENCE OF UKRAINE

National aerospace university “Kharkiv Aviation Institute”

Department of aircraft strength

Course
Mechanics of materials and structures

HOME PROBLEM 9

Stress Analysis of Two Supported Beams in Plane Bending

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“Kharkiv Aviation Institute”
Department of aircraft strength

Subject: mechanics of materials
Document: home problem
Topic: Stress Analysis of Two Supported Beams in plane Bending.

Full name of the student, group

Variant: 1 **Complexity: 1**

Given: $[\sigma]_t$ =160 MPa; $[\sigma]_c = 200$ MPa; $h/b = 2$ for rectangle cross-section.

Goal:
1) copy from home problem No5 the graphs of shear forces and bending moments ;

2) using condition of strength in pure bending calculate: a) diameter of round solid cross-section; b) diameters of hollow tube cross-section using thickness ratio $\alpha = d/D = 0{,}8$; c) dimensions of rectangle solid cross-section in $h/b = 2$; d) dimensions of hollow rectangle cross-section in $H/h = 2$; $B/b = 2$; e) number of I-beam section;

3) compare the weights of 5 cross-sections mentioned in p. 2;

4) design the graphs of acting stresses in cross-section with the largest shear force for 5 cross-sections mentioned in p.2;

5) estimate the type of stress state in the following points of I-beam section: a) lying on neutral axis; b) belonging to the most tensile or compressed layers of the section (choose yourself); c) in the point of the flange and web connection (one of two existing connections). Note, that the point must belong to the web.

Full name of the lecturer **signature**

Mark:

Given: $q = 10$ kN/m, $M = 20$ kNm, $P = 30$ kN, $a = 2$ m, $b = 4$ m, $c = 2$ m, $[s]_t = 160$ MПa, $[s]_c = 200$ MPa, $h/b = 2$.

Note 1. Stress analysis is possible only after calculation of internal forces in cross-sections of the beam.

Note 2. In shear force calculating, we will use the rule that internal shear force in particular cross-section numerically equals to algebraic sum of external forces projections on z axis, but only for the forces applied to the left or to the right part of the beam.

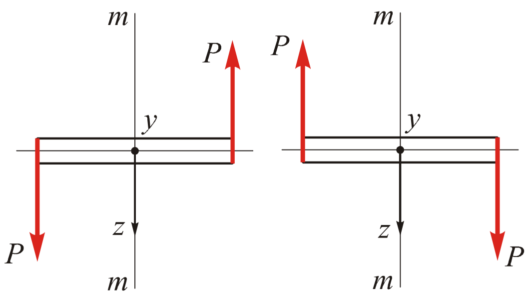
In bending moment calculating, we will use the rule that internal bending moment in particular cross-section numerically equals to algebraic sum of external force moments, but only for the forces and moments applied to the left or to the right part of the beam.

Solution

1. Sign conventions:

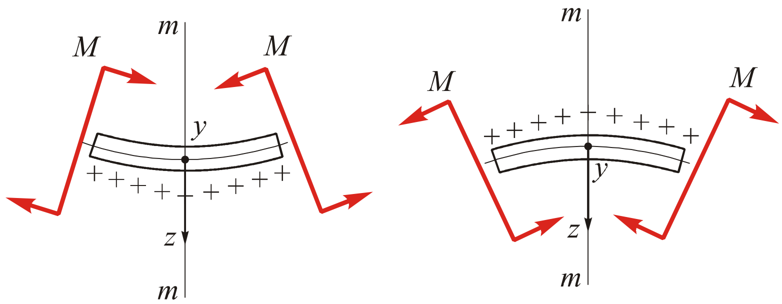
a) for shear forces (Fig. 1)

b) for bending moments (Fig. 2)



$Q_z^{m-m} < 0$ $Q_z^{m-m} > 0$

Fig. 1



$M_y^{m-m} > 0$ $M_y^{m-m} < 0$

Fig. 2

2. Calculating the reactions in supports R_A and R_C (see Fig. 3). Let us direct preliminary these reactions upwards, since their actual direction are unknown. Plus sign in solution will mean that really these reactions are directed upwards. Secondly, to determine R_A and R_C, we will use both equations of momentum balance (for example, relative to C and A points). Third equation of the force equilibrium will be used to check the result accuracy. In writing the equations of the moment balance, clockwise rotation will be assumed to be positive.

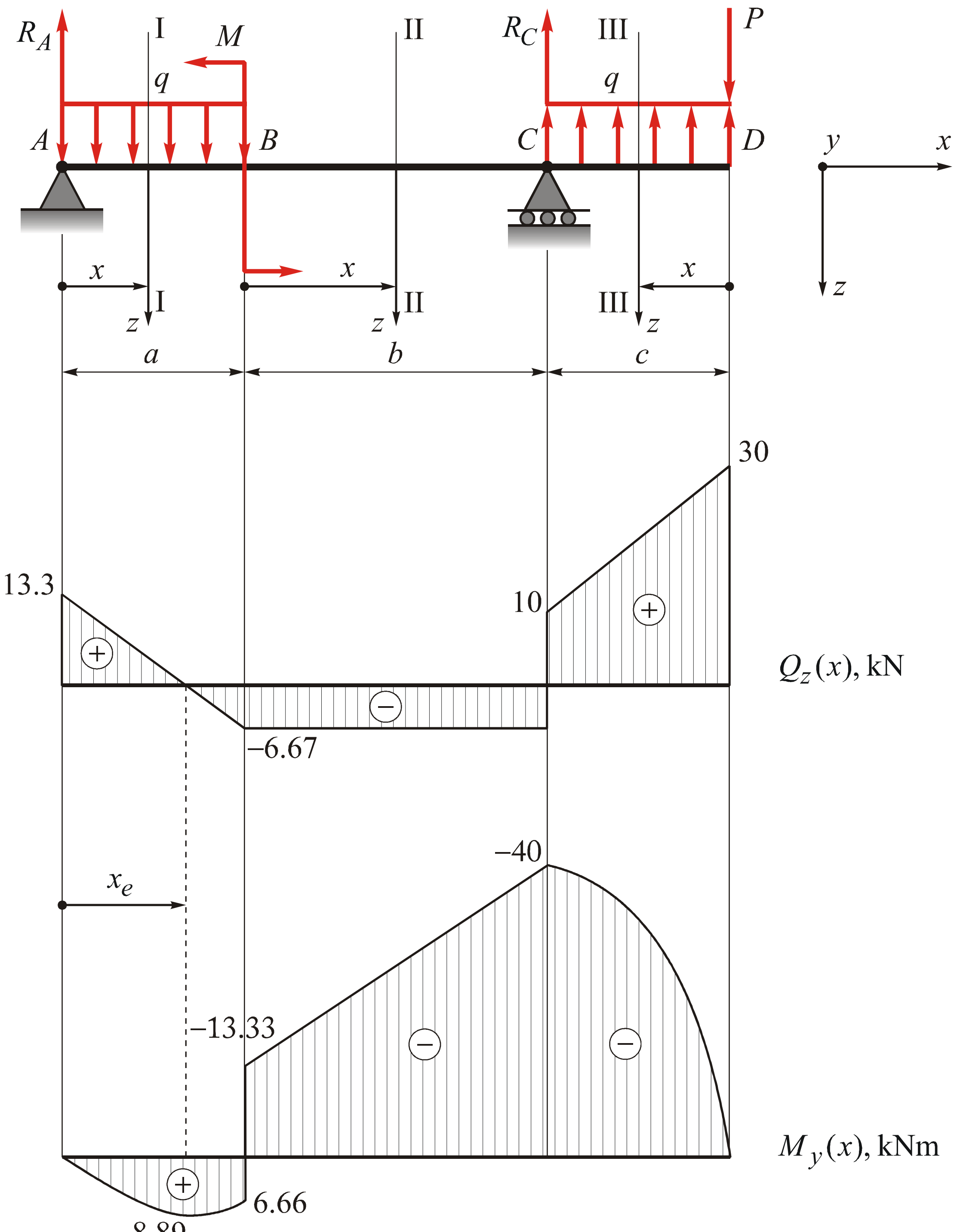


Fig. 3

$$\sum M_A = 0 = +\frac{qa^2}{2} - M - R_C(a+b) - qa\left(\frac{a}{2}+b+c\right) + P(a+b+c),$$

$$R_C = \frac{1}{a+b}\left(-\frac{qa^2}{2} + M + qa\left(\frac{a}{2}+b+c\right) - P(a+b+c) \right) = +16{,}67 \text{ kN}.$$

$$\sum M_C = 0 = -\frac{qc^2}{2} - M + R_A(a+b) - qa\left(\frac{a}{2}+b\right) + Pc,$$

$$R_A = \frac{1}{a+b}\left(+\frac{qc^2}{2} + M + qa\left(\frac{a}{2}+b\right) - Pc \right) = +13{,}33 \text{ kN}.$$

$$\sum P_z = 0 = -R_A - R_C - qc + qa + P = -13{,}33 - 16{,}67 - 10\times 2 + 10\times 2 + 30 = 0.$$

3. Determining the shear forces and bending moments in an arbitrary cross-sections of the beam. Two potions will be considered from the left and the last one from the right to get the simplest shape of equations.

I – I $\quad 0 < x < a$:

$$Q_z^I(x) = R_A - qx\,|_{x=0} = 13{,}33\,|_{x=2} = 13{,}33 - 20 = -6{,}67 \text{ kN},$$

$$M_y^I(x) = R_A x - \frac{qx^2}{2}\,|_{x=0} = 0\,|_{x=2} = 26{,}66 - 20 = 6{,}66 \text{ kNm}.$$

Note, that the change of shear force sign within this potion boundaries really means the presence of extreme value of internal bending moment within the boundaries of the potion. First of all, let us determine the coordinate of the cross-section with extreme bending moment. For this purpose, let us equate to zero the shear force equation:

$$Q_z^I(x_e) = 0 = R_A - qx_e = 13.33 - 10x_e, \quad x_e = 1.33 \text{ m (see Fig. 3)}.$$

Substituting this coordinate into bending moment equation leads to the following value:

$$M_{y_{\max}}^I = M_y^I(x_e) = R_A x_e - \frac{qx_e^2}{2} = 13.33 \times 1.33 - \frac{10}{2} \times 1.33^2 = +8{,}89 \text{ kNm}.$$

II – II $\quad 0 < x < b$:

$$Q_z^{II}(x) = R_A - qa = 13.33 - 20 = -6.67 \text{ kN},$$

$$M_y^{II}(x) = R_A(a + x) - qa\left(\frac{a}{2} + x\right) - M\,|_{x=0} = 26.66 - 20 - 20 =$$

$$= 13.34\,|_{x=4} = 79.98 - 100 - 20 = -40 \text{ kNm}.$$

III – III $\quad 0 < x < c$:

$$Q_z^{III}(x) = P - qx\,|_{x=0} = 30\,|_{x=2} = 30 - 20 = 10 \text{ kN},$$

$$M_y^{III}(x) = -Px + \frac{qx^2}{2}\,|_{x=0} = 0\,|_{x=2} = -60 + 20 = -40 \text{ kNm}.$$

4. Designing the shear force and bending moment graphs. For shear force graph positive values will be drawn upwards and vice versa. The bending moment graph will be drawn on tensile fibers (see Fig. 2).

In design problem solution, we will omit the shear forces due to their negligible influence on prismatic beam strength. In such case, we will determine critical section as the section with maximum magnitude of bending moment. In our problem, this cross-section is situated on the right support:

$$\left|M_{y_{\max}}\right| = 40 \text{ kNm}.$$

5. Calculating the sectional modulus $W_{n.a.}$ from condition of strength in critical cross-section:

$$s_{\max} = \frac{\left|M_{y_{\max}}\right|}{W_{n.a.}} \le [s].$$

Note. In the case when allowable stresses are different in tension and compression, lesser value of allowable stress should be used in condition of strength, i.e. $[s]_t = 160\ \text{MPa}$. Then

$$W_{n.a.} = \frac{\left|M_{y_{\max}}\right|}{[s]_t} = \frac{40 \cdot 10^3}{160 \cdot 10^6} = 250 \times 10^{-6}\ \text{m}^3.$$

6. Selecting the I-beam section number from the assortment.

(a) let us chose, at first, lesser number No.22 with $W_{n.a.} = 232 \times 10^{-6}\ \text{m}^3$:

$$\text{No.22} \rightarrow s_{\max} = \frac{40 \times 10^3}{232 \times 10^{-6}} = 172.4\ \text{MPa}.$$

This number will be evidently overstressed but five percent overstress is available in mechanics of materials. It's calculating shows, that

$$D = \frac{s_{\max} - [s]}{[s]} \times 100\% = \frac{172.4 - 160}{160} \times 100\% = 7.5\%.$$

Since overstress is more than 5%, No.22 is not applicable. Therefore, larger number should be selected: $\text{No.}22^a$ with $W_{n.a.} = 254 \times 10^{-6}\ \text{m}^3$. Maximum normal stress in this I-beam section is $s_{\max} = \dfrac{40 \times 10^3}{254 \times 10^{-6}} = 157.48\ \text{MPa}$.

For further calculation, copy from the assortment the following dimensions and geometrical properties of $\text{No.}22^a$ section:

$h = 22 \times 10^{-2}\ \text{m}$, $b = 12 \times 10^{-2}\ \text{m}$, $t = 0.89 \times 10^{-2}\ \text{m}$, $d = 0.54 \times 10^{-2}\ \text{m}$,

$I_y = 2790 \times 10^{-8}\ \text{m}^4$, $W_y = 254 \times 10^{-6}\ \text{m}^3$, $S_y^* = 143 \times 10^{-6}\ \text{m}^3$, $A^I = 32.8 \times 10^{-4}\ \text{m}^2$.

Note that *y*-axis is horizontal central axis for the section, which is really neutral axis in vertical bending. This section is shown on Fig. 4.

(b) design the graph of stress distribution in critical section under $\left|M_{y_{\max}}\right| = 40\ \text{kNm}$ and $\left|Q_z\right| = 10\ \text{kN}$ loading:

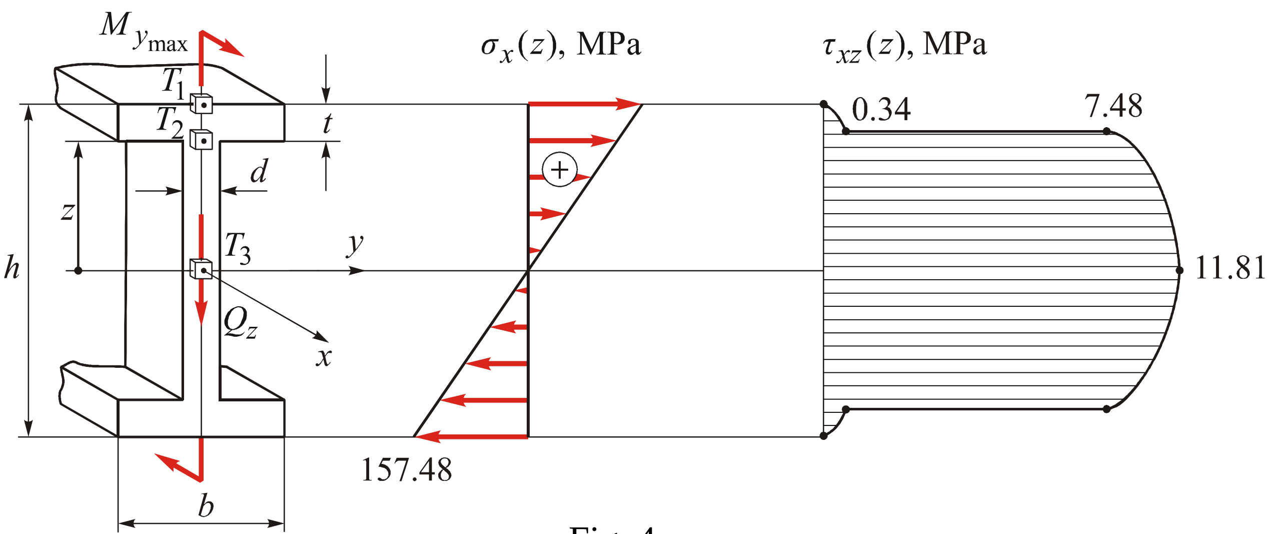


Fig. 4

To determine shear stresses and draw correspondent graph of their distribution, we will use the Juravsky formula. Knowing the stresses in three points: T_1 (outer point of the flange), T_2 (flange-web connection), T_3 (point of neutral axis), it becomes possible to draw parabolic graph of stress distribution.

$$t_1 = 0.$$

$$t_{2(flange)} = \frac{Q_z bt\left(\frac{h}{2}-\frac{t}{2}\right)}{bI_y} = \frac{10\times10^3\times12\times10^{-2}\times0.89\times10^{-2}\left(\frac{22\times10^{-2}}{2}-\frac{0.89\times10^{-2}}{2}\right)}{12\times10^{-2}\times2790\times10^{-8}} =$$

$$= 0.34 \text{ MPa}.$$

Note, that the flange width b was introduced into the Juravsky formula as the width of corresponding layer of the section.

$$t_{2(web)} = \frac{Q_z bt\left(\frac{h}{2}-\frac{t}{2}\right)}{dI_y} = \frac{10\times10^3\times12\times10^{-2}\cdot0.89\times10^{-2}\left(\frac{22\times10^{-2}}{2}-\frac{0.89\times10^{-2}}{2}\right)}{0.54\times10^{-2}\times2790\times10^{-8}} =$$

$$= 7.48 \text{ MPa}.$$

Note, that the web width d was introduced into the Juravsky formula as the width of corresponding layer of the section since this point belongs to the web.

$$t_3 = t_{\max} = \frac{Q_z S_y^*}{dI_y} = \frac{10\times10^3\times178\times10^{-6}}{0.54\times10^{-2}\times2790\times10^{-8}} = 11.81 \text{ MPa}.$$

Note, that the S_y^* value is the first moment of half-section relative to neutral axis of the section. It was preliminary found from assortment.

(c) analysis of the stress state type in T_1, $T_{2(web)}$, T_3 points of critical section (see Fig. 5, 6, 7):

Point T_1.

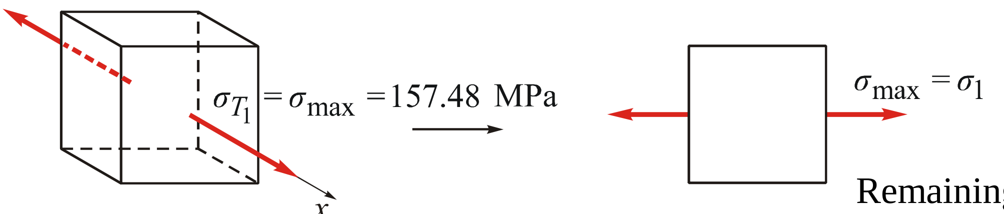


Fig. 5

Remaining principal stresses are: $s_2 = 0$, $s_3 = 0$

Conclusion 1: deformation is tension.
Conclusion 2: stress state is uniaxial.

Point $T_{2(web)}$.

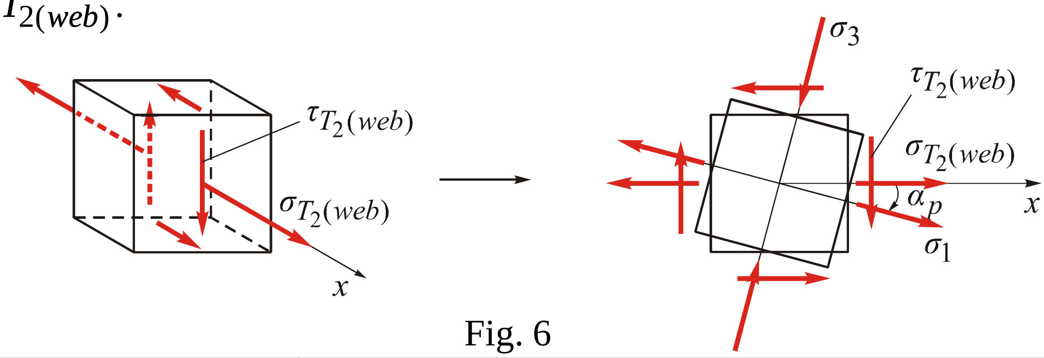


Fig. 6

$$s_{T_2(web)} = \frac{M_{y_{\max}}\left(\frac{h}{2}-t\right)}{I_y} = \frac{40\times10^3\left(\frac{22\times10^{-2}}{2}-0.89\times10^{-2}\right)}{2790\times10^{-8}} = 144.95\ \text{MPa},$$

$$t_{T_2(web)} = 7.48\ \text{MPa}.$$

To determine the stress-state type, let us determine principal stresses and the angle of principal planes inclination. The following formulae will be used for this purpose:

$$s_{\substack{\max\\ \min}} = \frac{s_a + s_b}{2} \pm \frac{1}{2}\sqrt{\left(s_a - s_b\right)^2 + 4t_a^2}\ ,$$

$$tg2a_p = \frac{2t_a}{s_b - s_a}.$$

Since the condition $s_a > s_b$ was assumed in these formulae proof, let us re-designate the stresses:

$$s_a = s_{T_2(web)} = +144.95\ \text{MPa}\,,$$

$$s_b = 0\,,$$

$$t_a = t_{T_2(web)} = +7.48\ \text{MPa}\,,$$

$$t_b = -t_a = -7.48\ \text{MPa}\,.$$

Then

$$s_{\substack{\max\\ \min}} = \frac{s_a + s_b}{2} \pm \frac{1}{2}\sqrt{\left(s_a - s_b\right)^2 + 4t_a^2} = \frac{144.95+0}{2} \pm \frac{1}{2}\sqrt{(144.95-0)^2 + 4\times7.48^2}\ .$$

$$s_{\max} = +145.34\ \text{MPa} = s_1,$$

$$s_{\min} = -0.39\ \text{MPa} = s_3,$$

Checking the invariability of normal stresses sum in rotation of axes:

$$s_a + s_b = s_1 + s_3 \ \rightarrow\ +144.95 + 0 = +145.34 - 0.39\,.$$

Calculation of the principal planes inclination:

$$\text{tg}2a_p = \frac{2t_a}{s_b - s_a} = \frac{2(+7.48)}{0-144.95} = -0.1032\,,$$

$$2a_p = -5.9° \ \rightarrow\ a_0 = -2.95°\ \text{(clockwise rotation)}.$$

Conclusion: stress state is plane (biaxial) (see Fig. 6).

Point T_3.

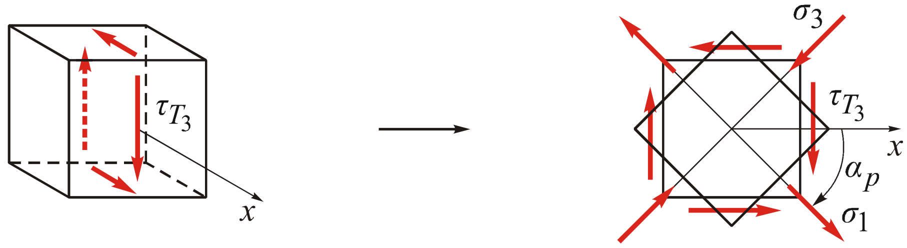


Fig. 7

In this case, $s_{T_3} = 0\ \text{MPa},\ t_{T_3} = t_{\max} = 11.81\ \text{MPa}.$

Conclusion: deformation type is pure shear, stress state is plane (biaxial) (see Fig. 7).

Calculation of principal stresses and the angle of principal planes inclination.

$$s_{\substack{max \\ min}} = \frac{s_a + s_b}{2} \pm \frac{1}{2}\sqrt{\left(s_a + s_b\right)^2 + 4t_a^2},$$

Since the condition $s_a > s_b$ was assumed in these formulae proof, let us re-designate the stresses:

$$s_a = s_b = 0,\ t_a = +t_{T_3} = +11.81\,\text{MPa},\ t_b = -t_a = -11.81\,\text{MPa}.$$

After substituting,

$$s_{max} = +t_a = +11.81\,\text{MPa} = s_1,$$

$$s_{min} = -t_a = -11.81\,\text{MPa} = s_3,$$

$$s_2 = 0.$$

Calculation of the principal planes inclination:

$$\text{tg}2a_p = \frac{2t_a}{s_b - s_a} = \frac{2(+11.81)}{0-0} = -\infty,\ 2a_p = -90°,\ a_p = -45°\ \text{(clockwise rotation)}$$

Principal stresses are shown on Fig. 7.

Conclusion: stress state is biaxial, deformation is pure shear.

7. Calculating the round cross-section diameter.

(a) it was found earlier from the condition of strength that $W_y = 250\times10^{-6}\ \text{m}^3$. On the other hand,

$$W_y^{\otimes} = \frac{pD^3}{32} \to D = \sqrt[3]{\frac{32W_y}{p}} = \sqrt[3]{\frac{32\times250\times10^{-6}}{3.14}} = 13.66\times10^{-2}\ \text{m}.$$

(b) cross-sectional area is

$$A^{\otimes} = \frac{pD^2}{4} = \frac{3.14(13.66\times10^{-2})^2}{4} = 146.48\times10^{-4}\ \text{m}^2.$$

Note, that this area is significantly more than the area of corresponding I-beam section: $A = 32.8\times10^{-4}\ \text{m}^2$**.**

(c) draw the graphs of stress distribution in critical section:

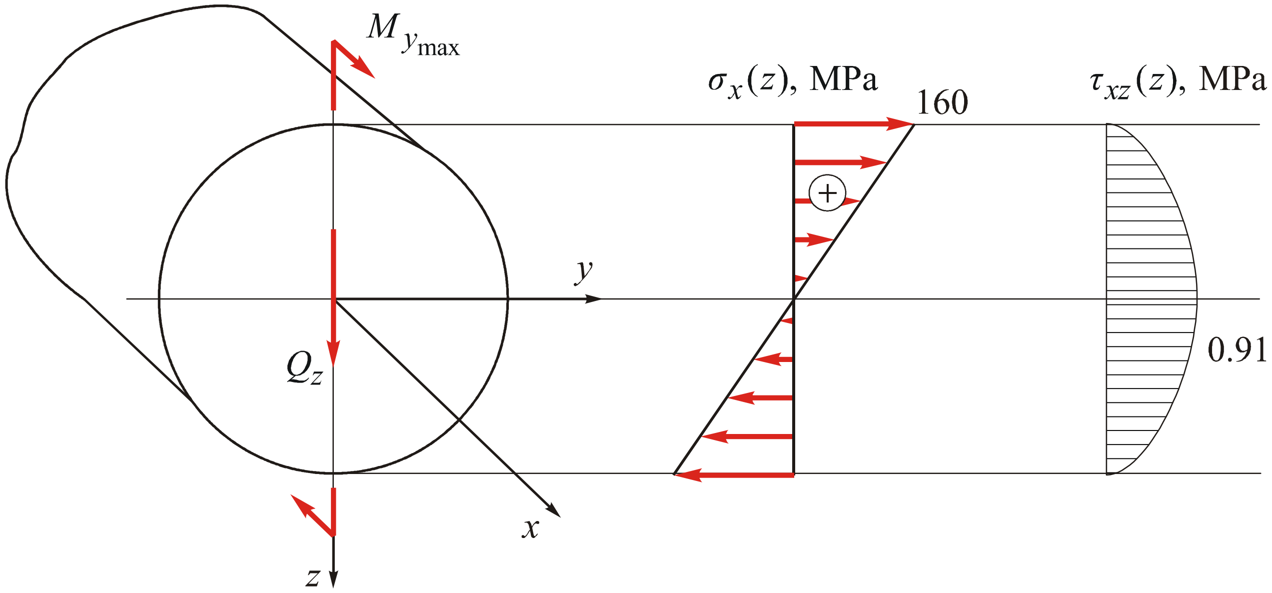


Fig. 8

$$s_{max} = \frac{32M_{y\max}}{pD^3} = \frac{32\times40\times10^3}{3.14\times(13.66\times10^{-2})^3} = 160\ \text{MPa},$$

$$t_{\max} = \frac{4}{3}\frac{Q_z}{A} = \frac{4\times 10\times 10^3}{3\times 146.48\times 10^{-4}} = 0.91 \text{ MPa}.$$

8. Calculating the dimensions for rectangle cross-section.
Let us assume, that $h/b = 2$.
(a) from condition of strength the sectional modulus should be equal to $W_y = 250\times 10^{-6}$ m^3. From the other hand, $W_y = \dfrac{bh^2}{6}$. After substituting $h = 2b$, we get

$$\frac{4b^3}{6} = 250\times 10^{-6} \text{ m}^3,$$

$$b \geq \sqrt[3]{\frac{3W_y}{2}} = \sqrt[3]{\frac{3\times 250\times 10^{-6}}{2}} = 7.22\times 10^{-2} \text{ m},$$

$$h = 14.44\times 10^{-2} \text{ m}.$$

(b) calculation of cross-section area:

$$A = bh = 104.26\times 10^{-4} \text{ m}^2.$$

Note, that this area is less than the area of round section but more than the area of I-beam section.

(c) draw the graphs of stress distribution in critical section:

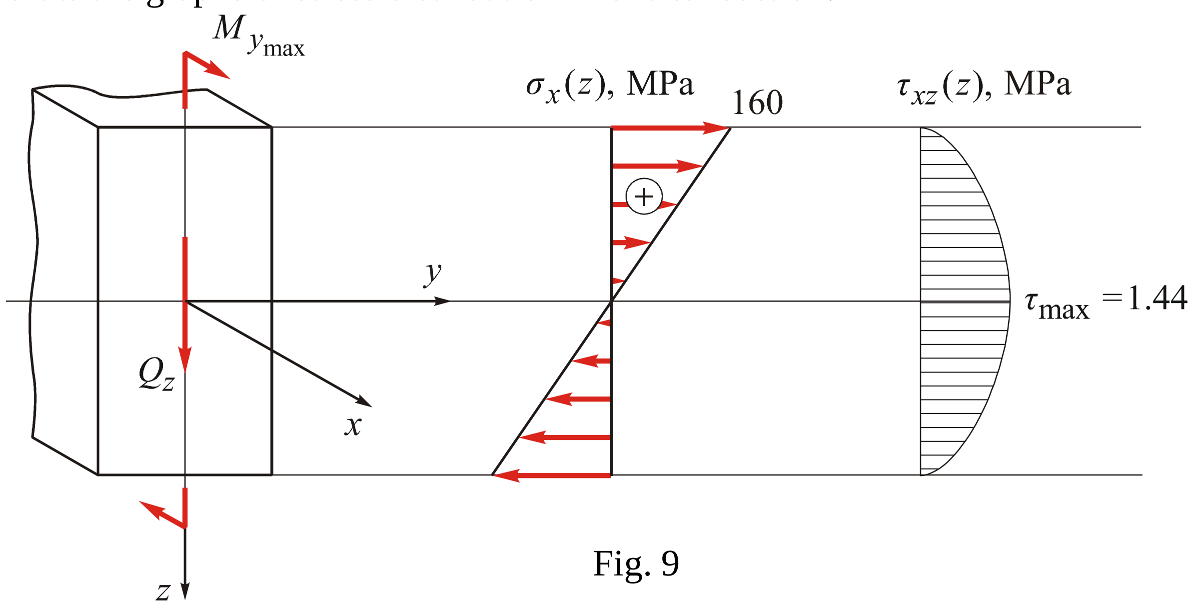


Fig. 9

$$s_{\max} = \frac{6M_{y\max}}{bh^2} = \frac{6\times 40\times 10^3}{7.22\times 10^{-2}\times\left(14.44\times 10^{-2}\right)^2} = 160 \text{ MPa},$$

$$t_{\max} = \frac{3}{2}\frac{Q_z}{A} = \frac{3\times 10\times 10^3}{2\times 104.26\times 10^{-4}} = 1.44 \text{ MPa}.$$

9. General conclusions:

a) $D^{\otimes} < h^{\mathrm{W}} < h^{\mathrm{I}}$ $(13.66\times 10^{-2}\text{ m} < 14.44\times 10^{-2}\text{ m} < 22\times 10^{-2}\text{ m})$;

b) $s^{\otimes}_{\max} = s^{\mathrm{W}}_{\max} \cong s^{\mathrm{I}}_{\max}$ $(160\text{ MPa} = 160\text{ MPa} \cong 157.48\text{ MPa})$;

c) $t^{\otimes}_{\max} < t^{\mathrm{W}}_{\max} = t^{\mathrm{I}}_{\max}$ $(0.91\text{ MPa} < 1.44\text{ MPa} = 11.81\text{ MPa})$;

d) $A^{\otimes} > A^{\mathrm{W}} > A^{\mathrm{I}}$ $(146.48\times 10^{-4}\text{ m}^2 > 104.26\times 10^{-4}\text{ m}^2 > 32.8\times 10^{-4}\text{ m}^2)$.