MINISTRY OF EDUCATION AND SCIENCE OF UKRAINE

National aerospace university “Kharkiv Aviation Institute”

Department of aircraft strength

Course
Mechanics of materials and structures

HOME PROBLEM 10

Stress Analysis of Rod System in Combined Loading

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National aerospace university
“Kharkiv Aviation Institute”
Department of aircraft strength

Subject: mechanics of materials
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Topic: Stress Analysis of the Rod System in Combined Loading.
Full name of the student, group

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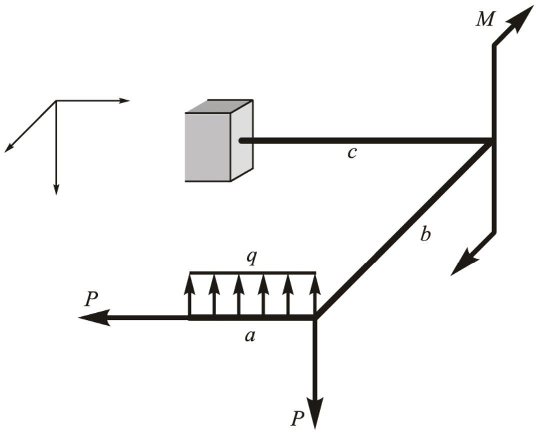


Given: $q = 10$ kN/m; $P = 20$ kN; $M = 10$ kNm; $[\sigma] = 160$ MPa; $a = 2$ m, $b = 3$ m; $c = 4$ m.
Goal:
1) write the equations of internal forces and moments in an arbitrary cross-sections of the rod system and draw the graphs of their distributions along the length of rod portions;

2) for the last portion: a) calculate the diameter of round solid cross-section; b) dimensions of rectangle solid cross-section in $h/b = 2$.

Full name of the lecturer **signature**

Mark:

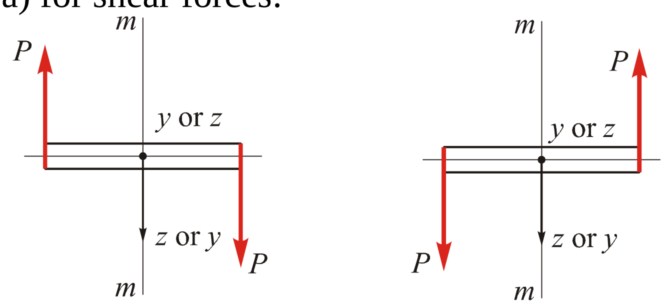
Given:
q=10kN/m,
P=20kN,
M=10kNm,
a=2m, b=3m, c=4m,
$[s]$=160MPa .

Goal:
(1) To write the equations of internal forces and moments in an arbitrary cross-sections of the rod system and draw the graphs of their distribution along the length of the rod.
(2) For the last portion:
a) calculate the diameter of round solid cross-section;
b) calculate dimensions of rectangle solid cross-section in $h/b = 2$.
Note. In this design problem solving, principle of superposition will be applied to estimate total effect of different internal forces on the stress state of potentially critical points of critical cross-section.

Solution

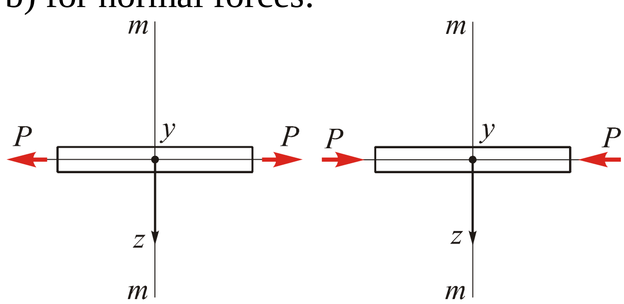
1. Sign conventions for internal forces and moments are the following (see Fig. 1):

a) for shear forces:



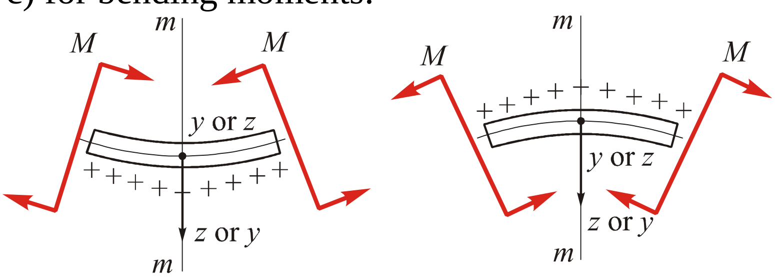
$Q_{z(y)}^{m-m} > 0$; $Q_{z(y)}^{m-m} < 0$;

b) for normal forces:



$N_x^{m-m} > 0$; $N_x^{m-m} < 0$;

c) for bending moments:



$M_{y(z)}^{m-m} > 0$; Fig. 1 $M_{y(z)}^{m-m} < 0$;

In the case of coincidence of the rod curvature with corresponding axis (y or z), the bending moment will be assumed as positive. The graphs of bending moments $M_y(x)$ generated by vertical forces will be drawn on tensile (+) fibers as well as bending moments $M_z(x)$ generated by horizontal forces.

2. Writing the equations of internal forces in an arbitrary cross-sections of the portions and drawing their graphs.

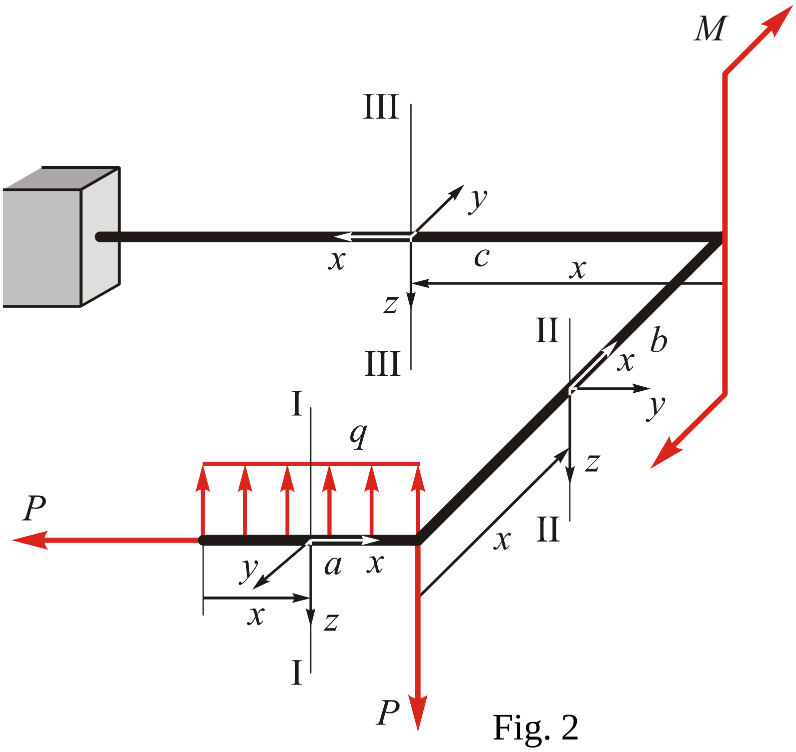


Fig. 2

I–I $\quad 0 < x < a, \qquad a = 2$ m

$N_x^I(x) = P = +20$ kN,

$M_x^I(x) = 0$ kNm,

$Q_z^I(x) = qx \,\Big|_{x=0} = 0 \text{ kN} \,\Big|_{x=2} = 20 \text{ kN},$

$M_y^I(x) = +qx\dfrac{x}{2} \,\Big|_{x=0} = 0 \text{ kN} \,\Big|_{x=2} = 20 \text{ kN},$

$Q_y^I(x) = 0$ kNm,

$M_z^I(x) = 0$ kNm.

II–II $\quad 0 < x < b, \qquad b = 3$ m

$N_x^{II}(x) = 0$ kN,

$M_x^{II}(x) = qa^2/2 = 20$ kNm,

$Q_z^{II}(x) = -P + qa = -20 + 20 = 0$ kN,

$M_y^{II}(x) = -Px + qax \,\Big|_{x=0} = 0 \text{ kNm} \,\Big|_{x=3} = 0 \text{ kNm},$

$Q_y^{II}(x) = -P = -20$ kN,

$M_z^{II}(x) = +Px \,\Big|_{x=0} = 0 \text{ kNm} \,\Big|_{x=3} = 60 \text{ kNm}.$

III–III $\quad 0 < x < c, \qquad c = 4$ m

$N_x^{III}(x) = -P = -20$ kN,

$$M_x^{III}(x) = M - Pb + qab = 10 \text{ kNm},$$
$$Q_z^{III}(x) = -P + qa = 0 \text{ kN},$$
$$M_y^{III}(x) = -Px + qax - qa\frac{a}{2}\Big|_{x=0} = -20 \text{ kNm} \quad \Big|_{x=4} = -80 + 80 - 20 = -20 \text{ kNm},$$
$$Q_y^{III}(x) = 0 \text{ kN},$$
$$M_z^{III}(x) = Pb = 60 \text{ kNm}.$$

3. Drawing the graphs (see Fig. 3):

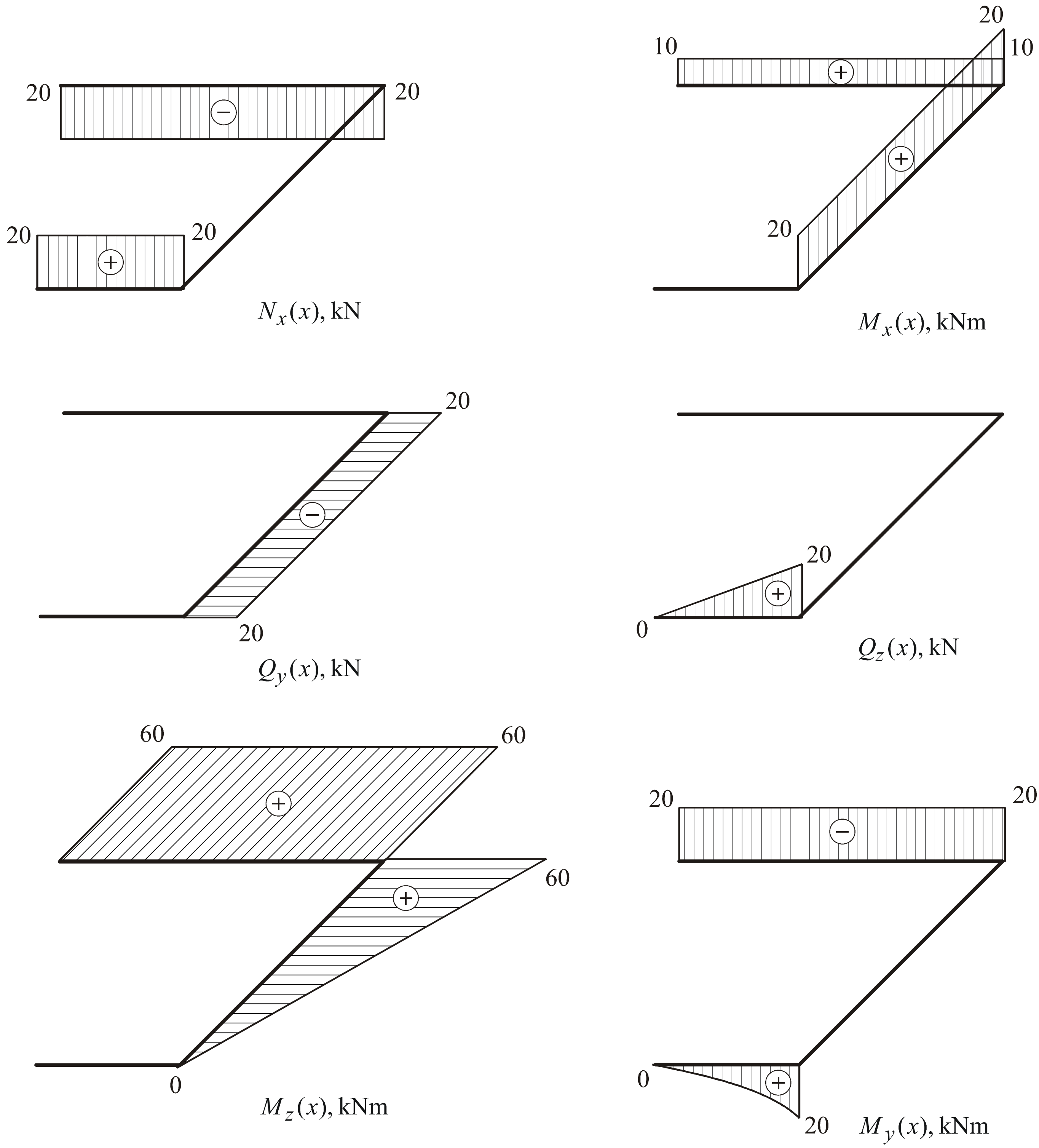


Fig. 3

4. Checking the solution, i.e. checking the equilibrium in the angular parts of the rod system (see Fig. 4).

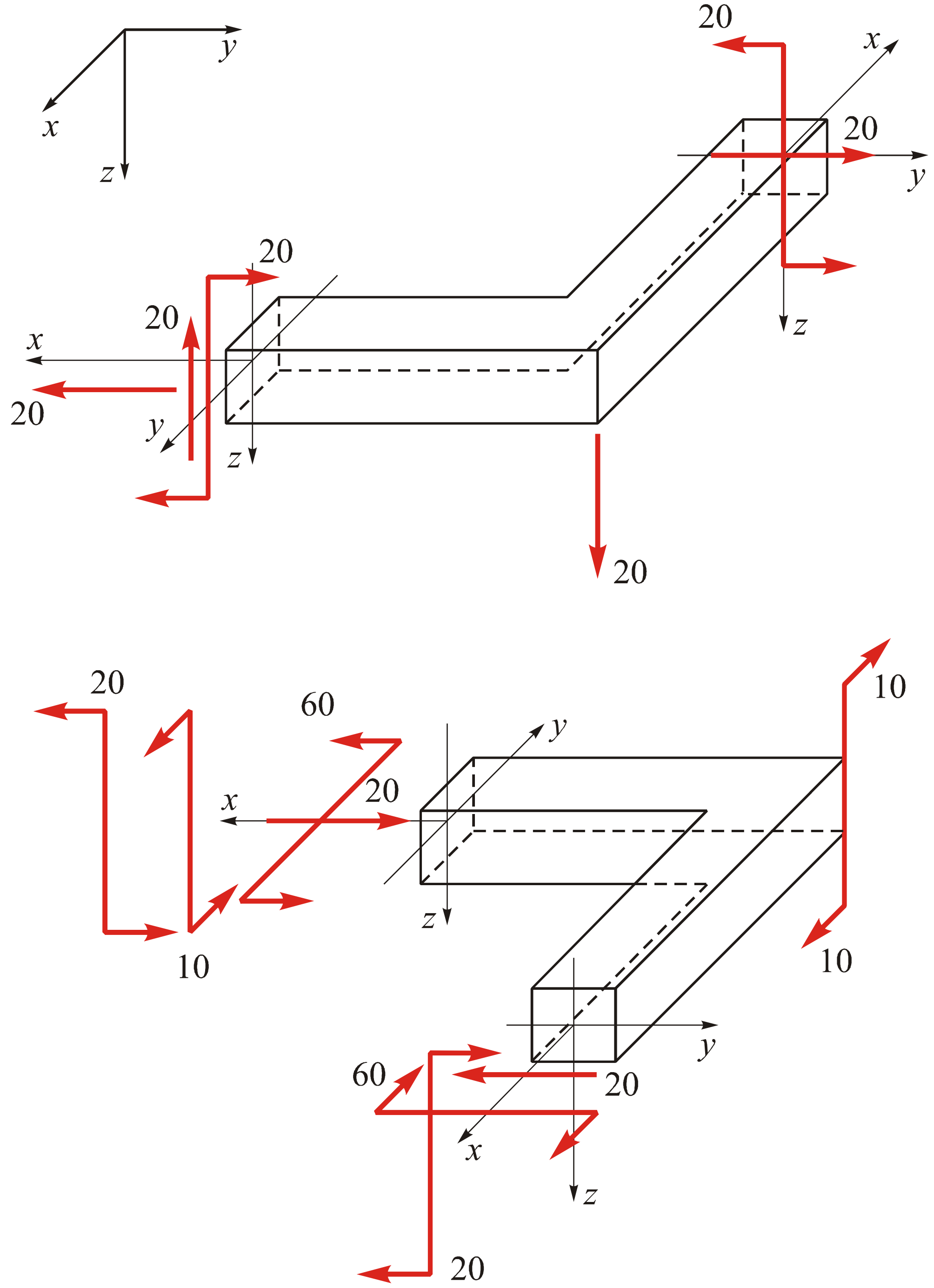


Fig. 4

Since the angles are in equilibrium, the internal forces are determined correctly.

5. Calculating the diameter of round solid cross-section for the last potion

First of all, let us determine the critical cross-section basing on the analysis of M_y and M_z moments distribution. In our case, all cross-sections are equicritical since internal forces are constant throughout the last portion length. After omitting shear forces Q_y and Q_z, remaining four internal forces are the following:

$$|N_x| = 20\ \text{kN},\ |M_x| = 10\ \text{kNm},\ |M_y| = 20\ \text{kNm},\ |M_z| = 60\ \text{kNm}.$$

They are applied to the critical section as shown in the Fig. 5:

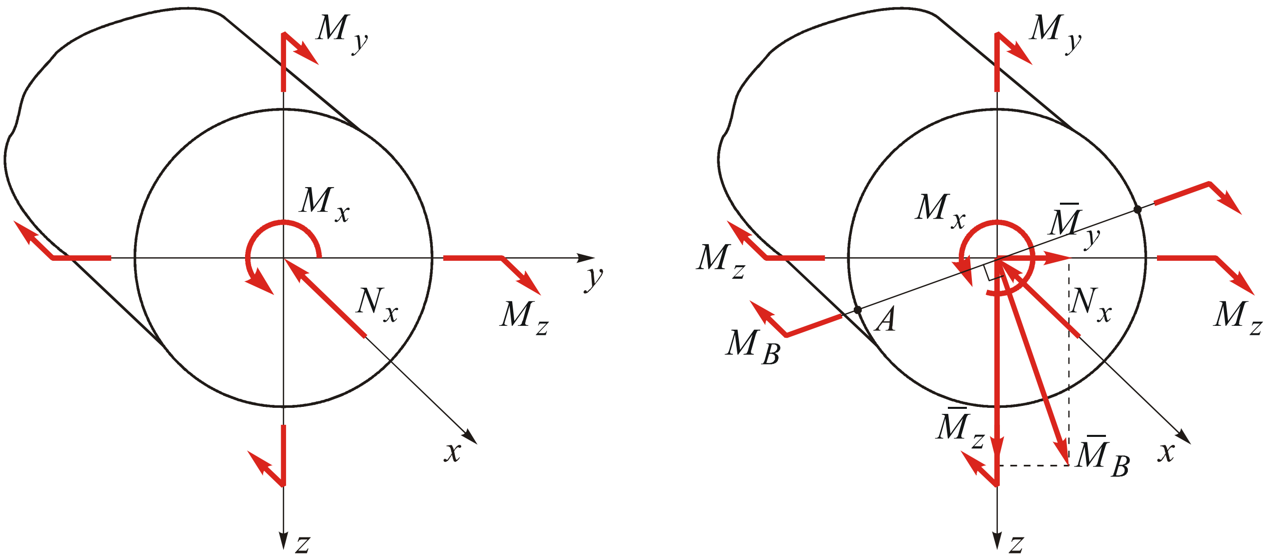


Fig. 5

Due to polar symmetry of cross-section it is possible to describe two bending moments M_y and M_z by resultant bending moment M_B and determine unique critical point A in the section:

$$M_B = \sqrt{M_y^2 + M_z^2} = \sqrt{20^2 + 60^2} = \sqrt{4000} = 63.245 \text{ kNm}.$$

Before writing the condition of strength in this point to solve design problem and determine the diameter it is necessary to estimate its stress state (see Fig. 6):

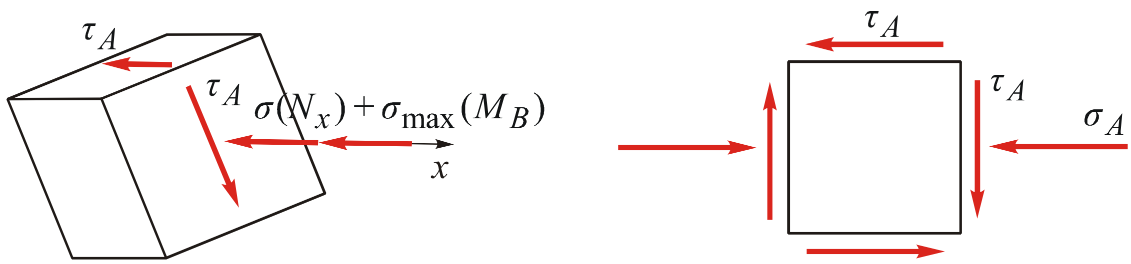


Fig. 6

$$\left| s_{p.A} \right| = \frac{N_x}{A} + \frac{M_B}{W_{n.a.}}, \qquad W_r = 2W_{n.a.}, \qquad t_{p.A} = \frac{M_x}{W_r} = \frac{M_x}{2W_{n.a.}}.$$

Since stress state is biaxial, we should use the corresponding strength theory, for example, the maximum shear stress theory:

$$s_{eq(p.A)}^{III} = \sqrt{s^2 + 4t^2} \le [s], \quad s_{eq(p.A)}^{III} = \sqrt{\left(\frac{N_x}{A} - \frac{M_B}{W_{n.a.}}\right)^2 + 4\left(\frac{M_x}{W_r}\right)^2} \le [s].$$

In the first approach, we will ignore $s\left(N_x\right)$. Simplified condition of strength becomes the following:

$$s_{eq(p.A)}^{III} = \sqrt{\frac{M_B^2}{W_{n.a.}^2} + 4\frac{M_x^2}{W_r^2}} \le [s], \text{ or } \frac{\sqrt{M_y^2 + M_z^2 + M_x^2}}{W_{n.a.}} \le [s].$$

Let us denote

$$M^S = \sqrt{M_y^2 + M_z^2 + M_x^2} = \sqrt{10^2 + 20^2 + 60^2} = 64.03 \text{ kNm}.$$

Resultant condition of strength is

$$s^{III}_{eq(p.A)} = \frac{M^S}{W_{n.a.}} \le [s].$$

$$W_{n.a.} \ge \frac{M^S}{[s]} \quad \rightarrow \quad D \ge \sqrt[3]{\frac{32\times 64.03\times 3\times 10^3}{3.14\times 160\times 10^6}} = 0.16\ \text{m}.$$

In the second approach, check the point A overstress using complete condition of strength:

$$s^{III}_{eq(p.A)} = \sqrt{\left(\left(\frac{4N_x}{pD^2}+\frac{32M_B}{pD^3}\right)^2+4\left(\frac{16M_x}{pD^3}\right)\right)^2} =$$

$$= \sqrt{\left(\frac{4\times 20\times 10^3}{3.14\times 0.16^2}+\frac{32\times 63.24\times 10^3}{3.14\times 0.16^3}\right)^2+4\left(\frac{16\times 10\times 10^3}{3.14\times 0.16^3}\right)^2} = 160.20\times 10^6\ \text{Pa} = 160.20\ \text{MPa}$$

.

Estimate the overstress:

$$Ds = \frac{s_{eq}-[s]}{[s]}\times 100\% = \frac{160.20-160}{160}\times 100 = 0.125\%.$$

So, the overstress is within the accepted limit of 5%.

6. Calculation of rectangle cross section dimensions.
(a) determining the potentially critical cross-sections.
In this case of loading, the internal forces are constant throughout the length of the last portion. So, in an arbitrary section,

$$|M_x| = 10\ \text{kNm},\ |M_y| = 20\ \text{kNm},\ |M_z| = 60\ \text{kNm},\ |N_x| = 20\ \text{kNm}.$$

Let us orient the cross section this way because M_z is larger than M_y to design more effective section in strength-to-weight ratio (see Fig. 7):

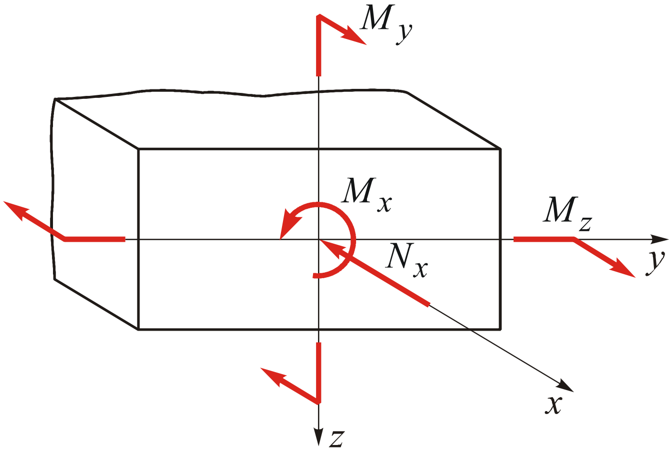


Fig. 7

(b) drawing the graphs of stress distributions (see Fig. 8):

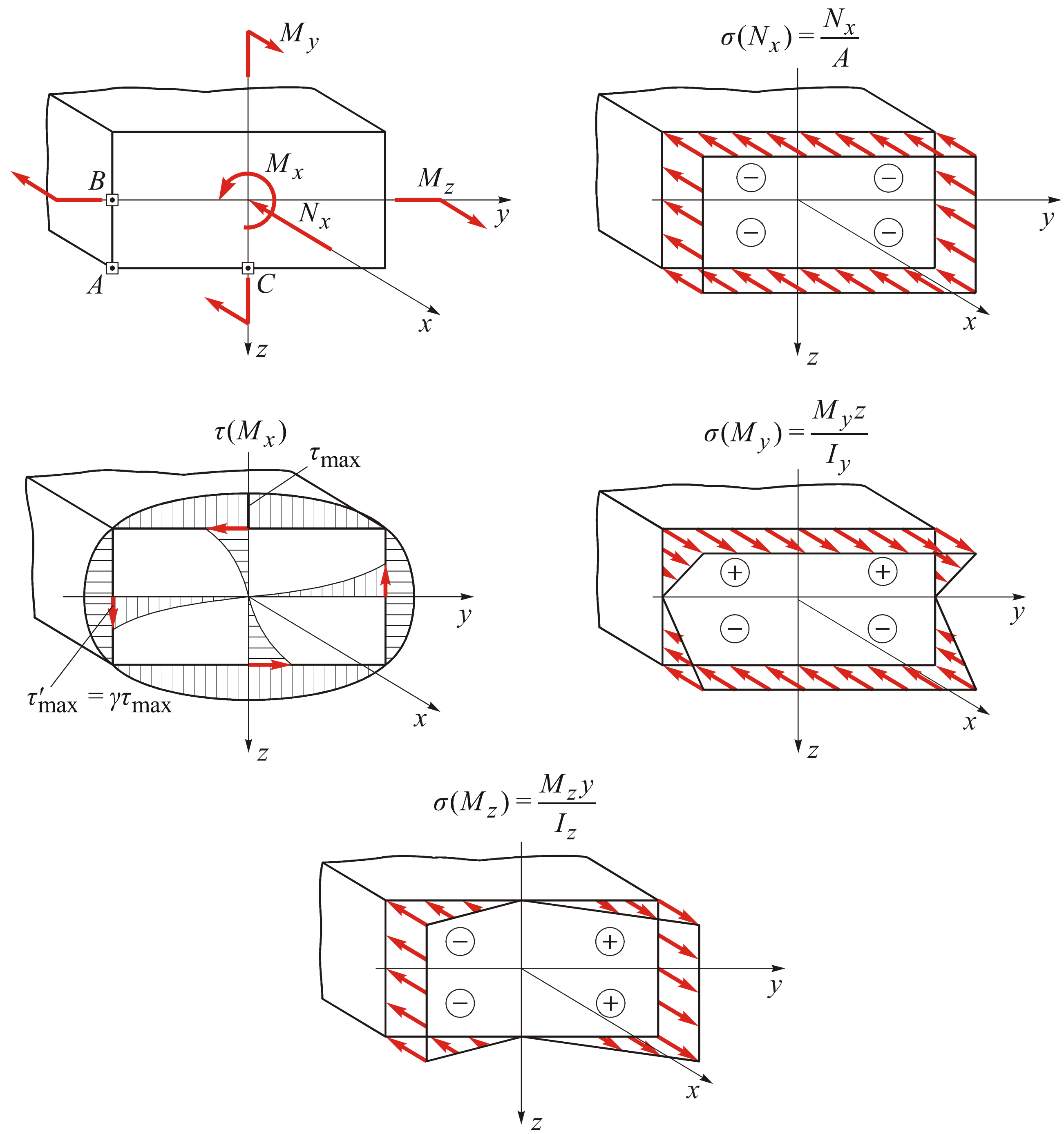


Fig. 8

(c) determining the potentially critical points.

In this case of loading, *A*, *B* and *C* points are potentially critical after analysis of stress distributions (see above). We will write the conditions of strength in these points and calculate three pairs of dimensions to select the largest pair.

(d) conditions of strength in potentially critical points of the section (see Figs 9, 10, 11).

Point *A*

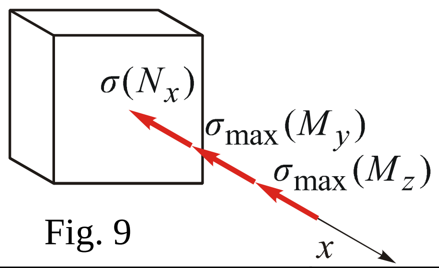


Fig. 9

$$|s_A| = \left|s\,(N_x) + s_{max}\left(M_y\right) + s_{max}\left(M_z\right)\right|,$$

$$t_A = 0.$$

Stress state is uniaxial and condition of strength is $|s_{max}| \le [s]$:

$$|s_{max}| = \frac{|N_x|}{A} + \frac{|M_y|}{W_y} + \frac{|M_z|}{W_z} \le [s],$$

$$\frac{N_x}{bh} + \frac{6M_y}{hb^2} + \frac{6M_z}{bh^2} = \frac{N_x}{2b^2} + \frac{3M_y}{b^3} + \frac{3M_z}{2b^3} \le [s].$$

In the first approach, we will ignore $s(N_x)$ and write simplified condition of strength:

$$\frac{3M_y}{b^3} + \frac{3M_z}{2b^3} \le [s], \quad b \ge \sqrt[3]{\frac{3M_y + 3M_z/2}{[s]}} = \sqrt[3]{\frac{3\times 20\times 10^3 + 3\times 60\times 10^3/2}{160\times 10^6}} =$$

$$= \sqrt[3]{0.9375\cdot 10^{-3}} = 0.098\ \text{m}.$$

In the second approach, we check the overstress at the point A:

$$|s_{max}| = \frac{N_x}{2b^2} + \frac{3M_y}{b^3} + \frac{3M_z}{2b^3} = (1041.233 + 64.75 + 95623.422)\times 10^3 =$$

$$= 161.41\times 10^6\ \text{Pa} = 161.41\ \text{MPa}.$$

Estimate the overstress

$$Ds = \frac{s_{max} - [s]}{[s]}\cdot 100\% = \frac{160.41 - 160}{160}\times 100\% = 0.26\%.$$

Conclusion: overstress is within the accepted limit of 5%.

Point *B*

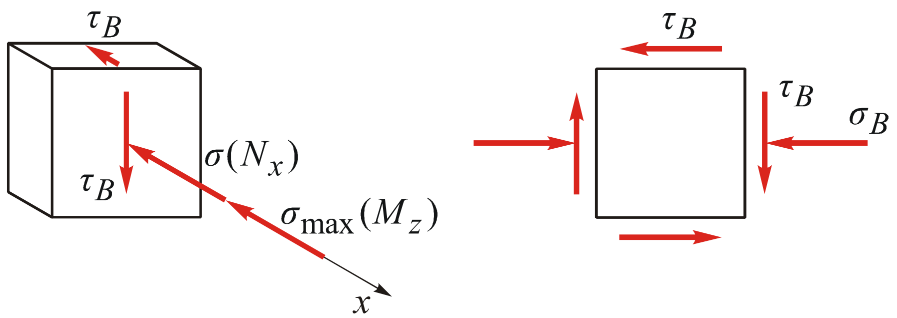


Fig. 10

$$|s_B| = \frac{N_x}{A} + \frac{M_z}{W_z} = \frac{N_x}{2b^2} + \frac{3M_z}{2b^3}, \qquad t_B = t'_{max} = \frac{g M_x}{W_t} = \frac{g M_x}{2a b^3}.$$

Stress state is plane and for condition of strength designing we will use corresponding strength theory.

$$s^{III}_{eq(p.B)} = \sqrt{s^2 + 4t^2} \le [s];$$

$$s^{III}_{eq(p.B)} = \sqrt{\left(\frac{N_x}{2b^2} + \frac{3M_z}{2b^3}\right)^2 + 4\left(\frac{g M_x}{2a b^3}\right)^2} \le [s].$$

In the first approach, we will ignore the $s\,(N_x)$ component:

$$b \ge \sqrt[3]{\frac{\sqrt{(3M_z/2)^2 + 4(g M_x/2a)^2}}{[s]}} = \sqrt[3]{\frac{\sqrt{\left(3\times60\times10^3/2\right)^2 + 4\left(0.795\times10\times10^3/(2\times0.246)\right)^2}}{160\times10^6}} =$$

$= 0.084$ m.

In the second approach, we estimate the point B overstress:

$$s_{eq(p.B)}^{III} = \sqrt{\left(\frac{N_x}{2b^2} + \frac{3M_z}{2b^3}\right)^2 + 4\left(\frac{g M_x}{2a b^3}\right)^2} =$$

$$= \sqrt{\left(\frac{20\times10^3}{2\times(0.084)^2} + \frac{3\times60\times10^3}{2\times(0.084)^3}\right)^2 + 4\left(\frac{0.795\times10\times10^3}{2\times0.246\times(0.084)^3}\right)^2} = \sqrt{2.65\times10^{16}} =$$

$= 162.67\times10^6$ Pa $= 162.67$ MPa.

Overstress $Ds = \dfrac{s_{eq(p.B)}^{III} - [s]}{[s]}\times100\% = 1.67\%$.

So, the overstress is within accepted 5% limit.

Point *C*

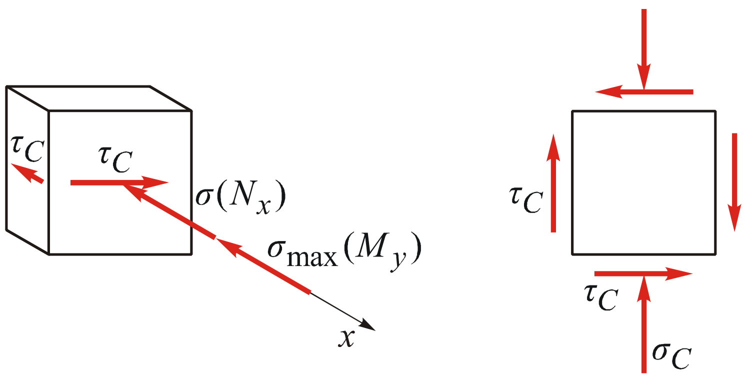


Fig. 11

$$|s_c| = \frac{|N_x|}{A} + \frac{|M_y|}{W_y} = \frac{N_x}{2b^2} + \frac{3M_y}{b^3}, \qquad t_c = t_{max} = \frac{M_x}{W_t} = \frac{M_x}{2a b^3}.$$

Stress state is plane and we will use corresponding strength theory to write condition of strength:

$$s_{eq(p.C)}^{III} = \sqrt{s^2 + 4t^2} \le [s],$$

$$s_{eq(p.C)}^{III} = \sqrt{\left(\frac{N_x}{2b^2} + \frac{3M_y}{b^3}\right)^2 + 4\left(\frac{M_x}{2a b^3}\right)^2} \le [s].$$

In the first approach, we will ignore $s\,(N_x)$ component to simplify the condition of strength. In result,

$$b \ge \sqrt[3]{\frac{\sqrt{(3M_y)^2 + 4(M_x/2a)^2}}{[s]}} = \sqrt[3]{\frac{\sqrt{\left(3\times20\times10^3\right)^2 + 4\left(10\times10^3/(2\times0.246)\right)^2}}{160\times10^6}} = 0.077 \text{ m}.$$

In the second approach, we estimate the overstress at point C applying full condition of strength:

$$s_{eq(p.C)}^{III} = \sqrt{\left(\frac{N_x}{2b^2} + \frac{3M_y}{b^3}\right)^2 + 4\left(\frac{M_x}{2ab^3}\right)^2} =$$

$$= \sqrt{\left(\frac{20\times10^3}{2\times(0.077)^2} + \frac{3\times20\times10^3}{(0.077)^3}\right)^2 + 4\left(\frac{10\times10^3}{2\times0.246\times(0.077)^3}\right)^2} =$$

$$= \sqrt{2.565\times10^{16}} = 160.15\times10^6 \text{ Pa} = 160.15 \text{ MPa}.$$

Overstress $Ds = \dfrac{s_{eq(p.C)}^{III} - [s]}{[s]}\times100\% = \dfrac{160\times15-160}{160}\times100\% = 0.09\%$.

So, the overstress is within the accepted 5% limit.

(e) selecting the largest pair of dimensions.

The values of b & h from the three potentially critical points are the following:

$p.A$ $b = 0.098$ m, $h = 0.196$ m,
$p.B$ $b = 0.084$ m, $h = 0.168$ m,
$p.C$ $b = 0.077$ m, $h = 0.154$ m.

We choose the largest values, so $b = 0.098$ m and $h = 0.196$ m.

5. Comparing the rectangle dimensions with the round section diameter.

(a) the diameter of round solid cross section is 0.16 m.

(b) the h & b of solid rectangle cross section are $h = 0.196$ m, b=0.098 m.

Conclusion: $h > D$ $(0.196 > 0.16)$.