MINISTRY OF EDUCATION AND SCIENCE OF UKRAINE

National aerospace university “Kharkiv Aviation Institute”

Department of aircraft strength

Course
Mechanics of materials and structures

HOME PROBLEM 11

Stress Analysis of Beams in Oblique Bending

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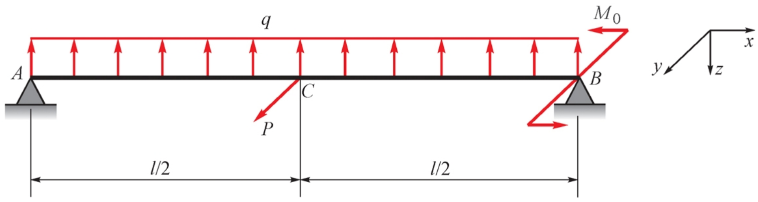
“Kharkiv Aviation Institute”
Department of aircraft strength

Subject: mechanics of materials
Document: home problem
Topic: Stress Analysis of the Beam in Oblique Bending.

Full name of the student, group

Variant: 3 **Complexity: 1**



Given: $q = 10$ kN/m; $P = 10$ kN; $M_0 = 20$ kNm; $[\sigma] = 160$ MPa; $a = 2$ m, $c = 4$ m. Cross-section: a) rectangle (h=20cm, b=10cm); b) I-beam №___.

Goal:
1) draw the graphs of bending moments $M_y(x)$ and $M_z(x)$;
2) design the graph of stress distribution in critical cross-section;
3) find critical point in critical section and estimate the strength of the beam;
4) analytically find position of neutral axis in critical cross-section.

signature

Full name of the lecturer

Mark:

Is given: the simply supported I-beam №20 with two axes of cross-sectional symmetry and the length $l = 2$ m, under external oblique loading generated by horizontal and vertical forces and moments: $P = 10kN$, $q = 10$kN/m, $M_0 = 20$ kNm.

It is necessary:
1. To determine the diagrams of internal forces and find critical section of oblique bending.
2. To construct the diagrams of the stress distribution in critical section of oblique bending and determine the neutral axis orientation graphically.
3. To determine the neutral axis orientation analytically.

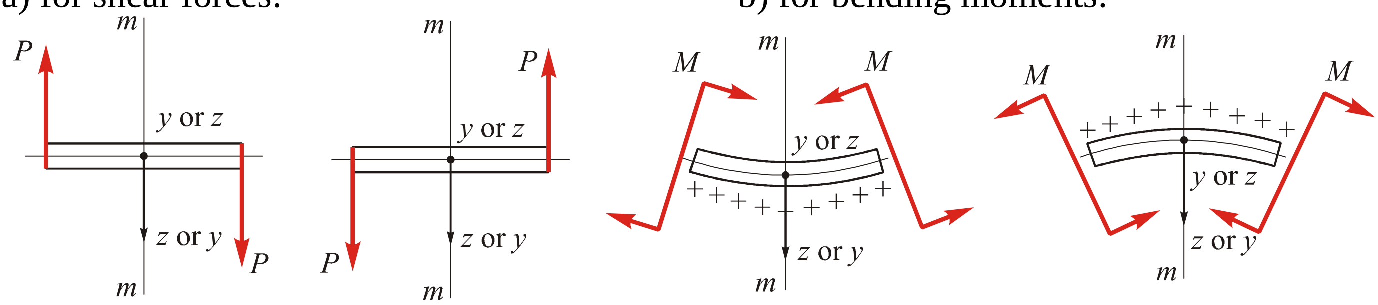
Note 1. Stress analysis is available only after calculation of internal forces in cross-sections of the beam.

Note 2. In Q_y and Q_z shear force calculating, we will use the rule that internal shear force in particular cross-section numerically equals to algebraic sum of external forces projections on y or z axis, respectively, but only for the forces applied to the left or to the right part of the beam.

In M_y and M_z bending moment calculating, we will use the rule that internal bending moment in particular cross-section numerically equals to algebraic sum of the moments generated by vertical or horizontal external forces, respectively, but only by external forces and moments applied to the left or to the right part of the beam.

Note 3. We will use the sign conventions which were assumed earlier in designing the internal forces graphs under combined loading:

a) for shear forces: b) for bending moments:



$Q_z^{m-m} > 0$ or $Q_y^{m-m} > 0$; $Q_z^{m-m} < 0$ or $Q_y^{m-m} < 0$; $M_y^{m-m} > 0$ or $M_z^{m-m} > 0$; $M_y^{m-m} < 0$ or $M_z^{m-m} < 0$

Fig. 1

Solution

1. Designing the graphs of Q_z, M_y and Q_y, M_z functions.

(a) calculation of the reactions in supports: vertical reactions R_{A_v}, R_{B_v} and R_{A_h}, R_{B_h}. Originally they are directed upwards.

Using the equilibrium equations in vertical plane we determine R_{A_v}, R_{B_v}:

$$\sum M_A = 0, \quad R_{B_v} l + ql\frac{l}{2} = 0 \rightarrow R_{B_v} = -\frac{ql^2}{2l} = -\frac{10\times 2^2}{2\times 2} = -10\ \text{kN}.$$

"–" sign shows that R_{B_v} acts in opposite direction, i.e. it's actual direction is downwards. So, we change the original direction of R_{B_v} action on opposite in Fig. 2.

$$\sum M_B = 0, \quad -R_{A_v} l - \frac{ql^2}{2} = 0 \rightarrow R_{A_v} = -\frac{ql^2}{2l} = -10\ \text{kN}.$$

And we change the direction of R_{A_v} action into the opposite one once again.

Let us check up the balance after calculations equating to zero all forces in vertical plane:

$$\sum F_z = 0, \quad R_{A_v} + R_{B_v} - ql = 10 + 10 - 20 = 0.$$

Using the equilibrium equations in horizontal plane, we will determine R_{A_h}, R_{B_h}:

$$\sum M_A = 0, \quad -R_{B_h} l - \frac{Pl}{2} + M_0 = 0 \rightarrow R_{B_h} = \frac{P\frac{l}{2} - M_0}{l} = \frac{10\times\frac{2}{2} - 20}{2} = -5\ \text{kN}.$$

Due to "–" sign of R_{B_h}, let us change the original direction of R_{B_h} on opposite (see Fig. 2).

$$\sum M_B = 0, \quad -R_{A_h} l + \frac{Pl}{2} + M_0 = 0 \rightarrow R_{A_h} = \frac{P\frac{l}{2} + M_0}{l} = \frac{10\times\frac{2}{2} + 20}{2} = +15\ \text{kN}.$$

Let us check up the balance in horizontal plane:

$$\sum F_y = 0, \quad -R_{A_h} + R_{B_h} + P = -15 + 5 + 10 = 0.$$

(b) Equations of internal forces:

Portion *I-I*: $0 < x < l/2$ (see Fig. 2)

$$Q_z^I(x) = R_{B_v} - qx\Big|_{x=0} = 10\Big|_{x=1} = 0\ \text{kN}, \qquad M_y^I(x) = -R_{B_v}x + \frac{qx^2}{2}\Big|_{x=0} = 0\Big|_{x=1} = -5\ \text{kNm},$$

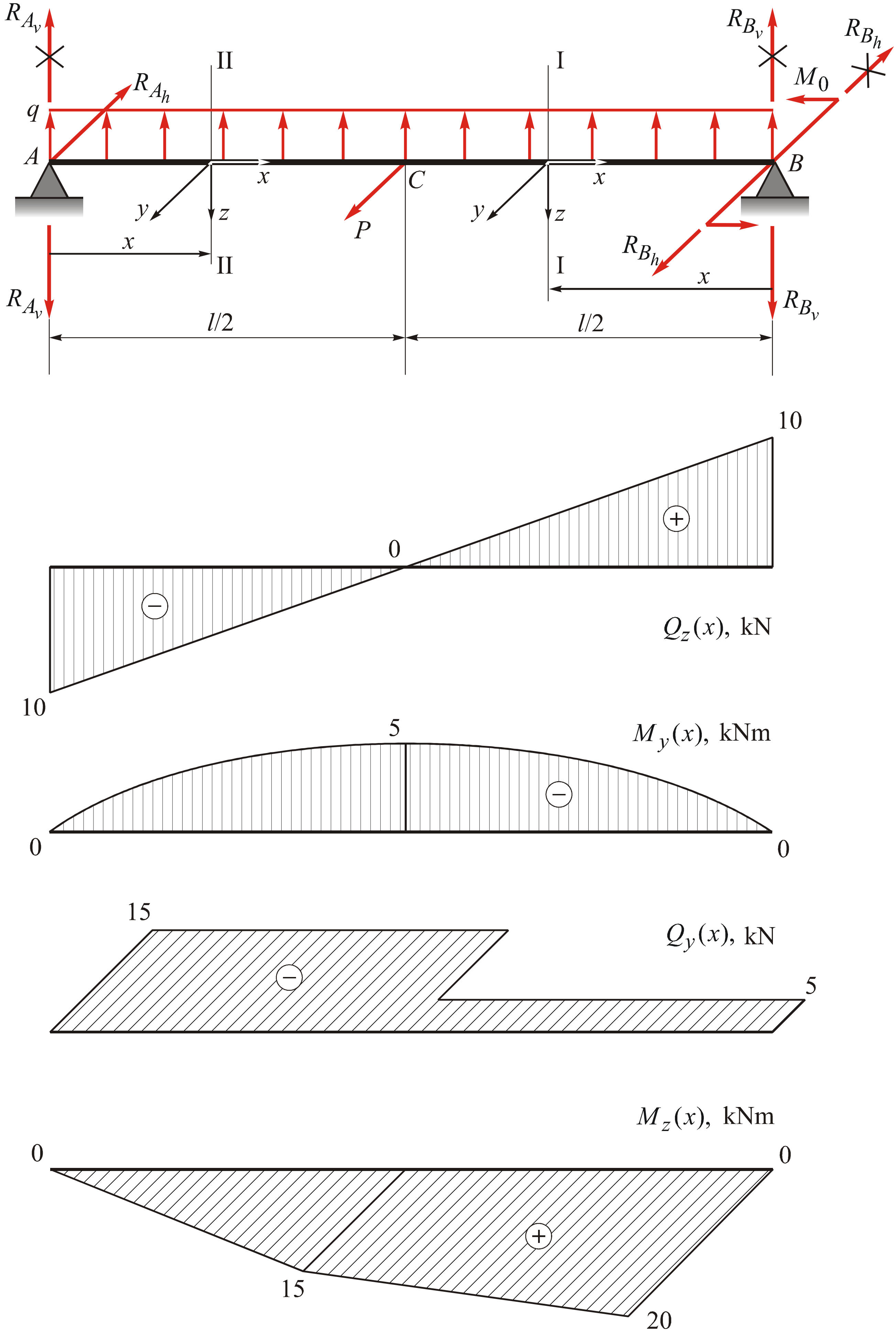
$$Q_y^I(x) = -R_{B_h} = -5\ \text{kN}, \qquad M_z^I(x) = -R_{B_h}x + M_0\Big|_{x=0} = 20\Big|_{x=1} = +15\ \text{kNm}.$$

Portion *II-II*: $0 < x < l/2$ (see Fig. 2)

$$Q_z^{II}(x) = -R_{A_v} + qx\Big|_{x=0} = -10\Big|_{x=1} = 0\ \text{kN}, \quad M_y^{II}(x) = -R_{A_v}x + \frac{qx^2}{2}\Big|_{x=0} = 0\Big|_{x=1} = -5\ \text{kNm},$$

$$Q_y^{II}(x) = -R_{A_h} = -15\ \text{kN}, \qquad M_z^{II}(x) = +R_{A_h}x\Big|_{x=0} = 0\Big|_{x=1} = +15\ \text{kNm}.$$

Now let us construct the internal force factors diagrams in order to find $|M_y|_{\max}$ and $|M_z|_{\max}$ and also estimate potentially critical cross-sections of oblique bending. For this purpose, we analyze the graphs of M_y and M_z distributions (see Fig. 2).



R_{A_v}
R_{A_h}
R_{B_v}
R_{B_h}
M_0
q
A
B
C
P
II
I
x
y
z
l/2
$Q_z(x)$, kN
$M_y(x)$, kNm
$Q_y(x)$, kN
$M_z(x)$, kNm
10
0
5
15
20

Fig. 2

The most critical combination of two bending moments takes place in middle cross-section C with $|M_y|_{max} = 5$ kNm and $|M_z| = 15$ kNm. Also, B support is potential critical in horizontal bending under $|M_z| = 20$ kNm.

2. Calculating acting stresses in critical cross-section and graphical finding of the neutral axis orientation.

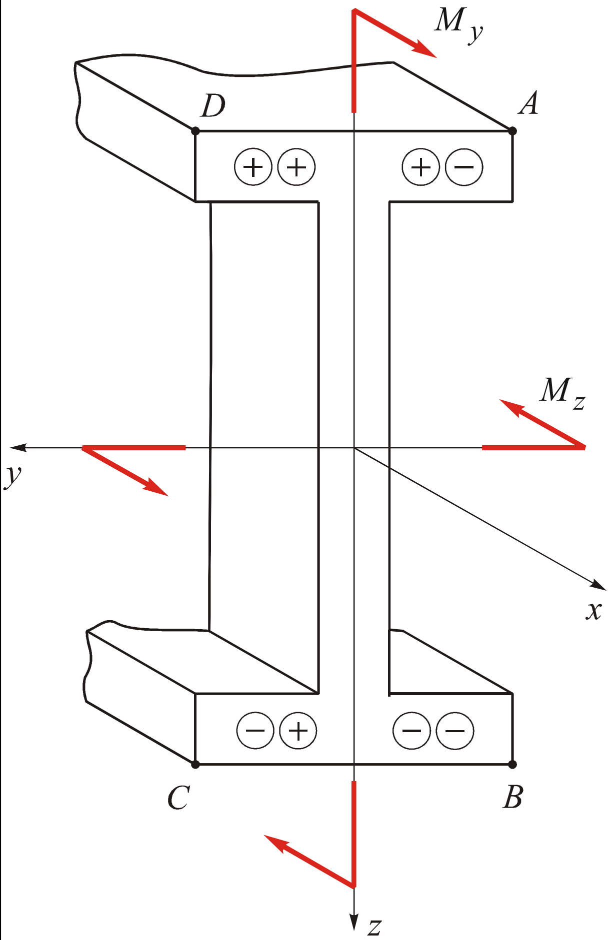


Fig. 3

Draw spatial view of critical cross-section in scale and apply the bending moments (see Fig. 3).

Taking into consideration the fact that the number of the given I-beam is 20 and using the assortment, we put down the values of its geometrical properties:

$I_y = 1840\ \text{cm}^4$ (maximal principal value) and $W_y = 184\ \text{cm}^3$,

$I_z = 115\ \text{cm}^4$ (minimal principal value) and $W_z = 23.1\ \text{cm}^3$,

$h = 20$ cm, $b = 10$ cm, $s = 0.52$ cm (web), $t = 0.84$ cm (flange).

In order to construct the stress distribution across the I-beam cross section, we calculate maximum values of stresses for the points *A*, *B*, *C*, *D*.

$$s_A = \frac{M_y}{W_y} - \frac{M_z}{W_z} = \frac{5\times10^3}{184\times10^{-6}} - \frac{15\times10^3}{23.1\times10^6} = -622\ \text{MPa},$$

$$s_B = -\frac{M_y}{W_y} - \frac{M_z}{W_z} = -\frac{5\times10^3}{184\times10^{-6}} - \frac{15\times10^3}{23.1\times10^6} = -676\ \text{MPa},$$

$$s_C = -\frac{M_y}{W_y} + \frac{M_z}{W_z} = -\frac{5\times10^3}{184\times10^{-6}} + \frac{15\times10^3}{23.1\times10^6} = +622\ \text{MPa},$$

$$s_D = \frac{M_y}{W_y} + \frac{M_z}{W_z} = +\frac{5\times10^3}{184\times10^{-6}} + \frac{15\times10^3}{23.1\times10^6} = +676\ \text{MPa}.$$

The calculations, mentioned above, are necessary to draw the graph of stress distribution over the cross-section contour. It is shown on Fig. 4. Neutral axis position is determined by the intersection of stress distribution graph with the cross-section contour *ABCD* (*EF* line on Fig. 4).

3. Analytical determining the neutral axis orientation.
In general case, the equation of the neutral axis is determined by equating to zero the stress in an arbitrary point *K* with coordinates *y* and *z* (see Fig. 5):
$s_K = -\frac{M_y z}{I_y} + \frac{M_z y}{I_z} = 0$, or $z = +\frac{M_z}{M_y}\frac{I_y}{I_z} y$, or $z = +ky$, where the slope of neutral axis inclination is $k = \tan a$:

$$\tan a_p = \frac{M_z}{M_y}\frac{I_y}{I_z} = \left(\frac{15\times10^3}{5\times10^3}\times\frac{1840\times10^{-8}}{115\times10^{-8}}\right) = 48.0\text{, where } a_p = +88^{\mathbf{0}}.$$

This positive inclination of neutral axis in *yOz* system of coordinates is shown on Fig. 5. It is coincident with graphical solution shown on Fig. 4.

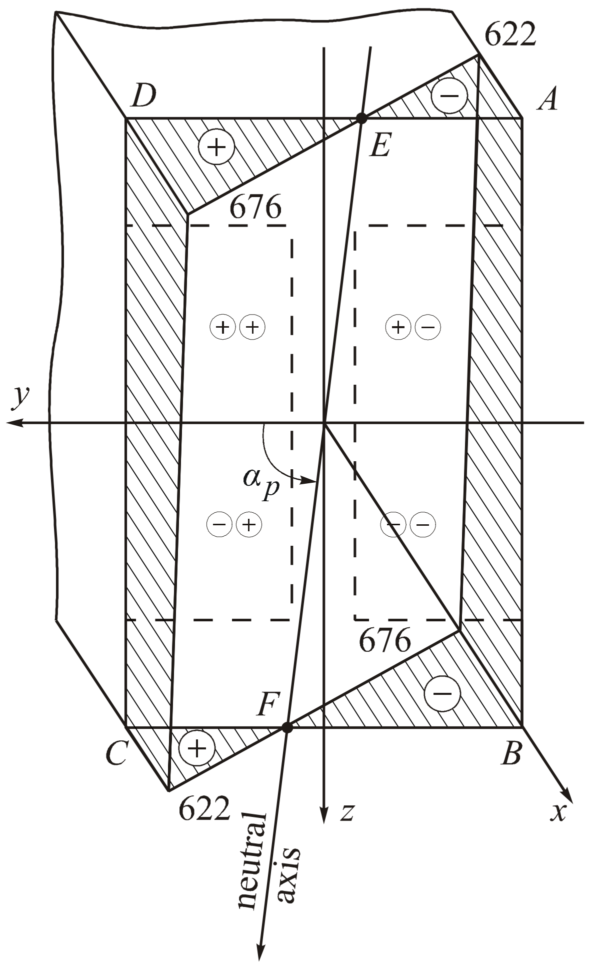


Fig. 4

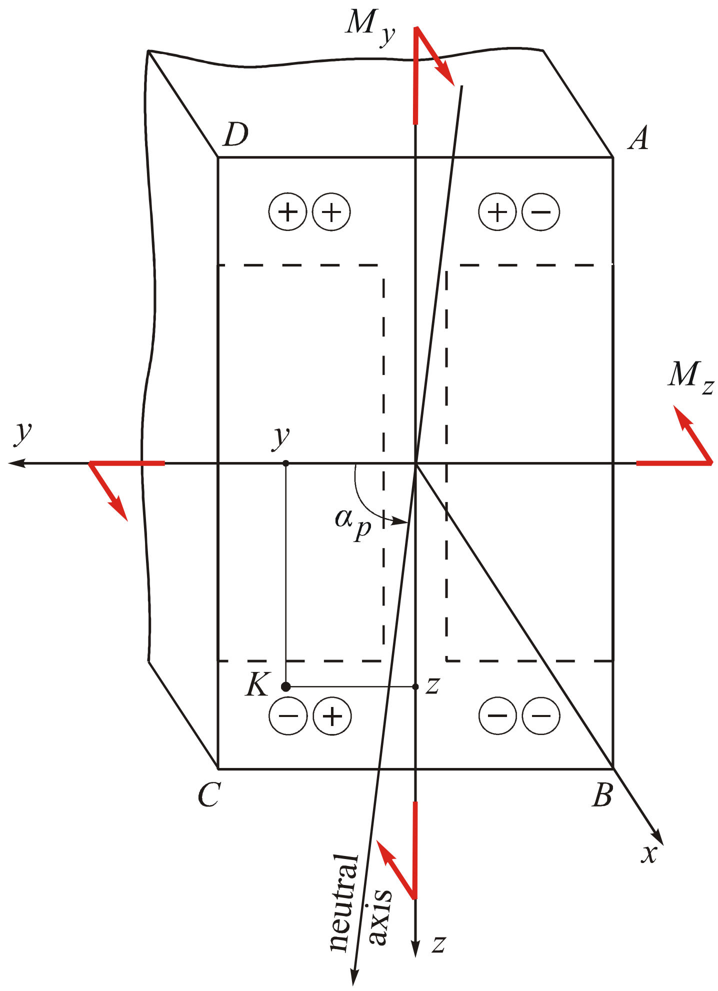


Fig. 5

4. To perform full scale stress analysis of the beam, let us consider the section *B* (right support), where only one maximum bending moment $|M_z| = 20$ kNm acts in horizontal plane, and $M_y = 0$ (see Fig. 6). Let us construct the stress distribution diagram after calculating acting stresses and evaluate maximal stresses in the beam:

$$s_{\max A,B} = -\frac{M_z}{W_z} = -\frac{20\times10^3}{23.1\times10^6} = -865\text{ MPa},$$

$$s_{\max C,D} = +\frac{M_z}{W_z} = +\frac{20\times10^3}{23.1\times10^6} = +865\text{ MPa}.$$

Neutral axis is coincident with *z* direction.

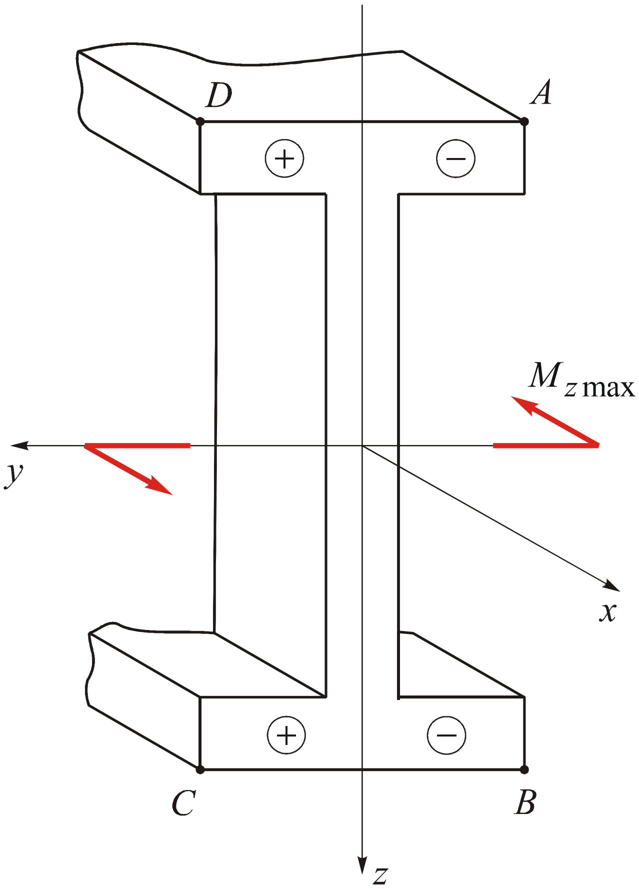


Fig. 6

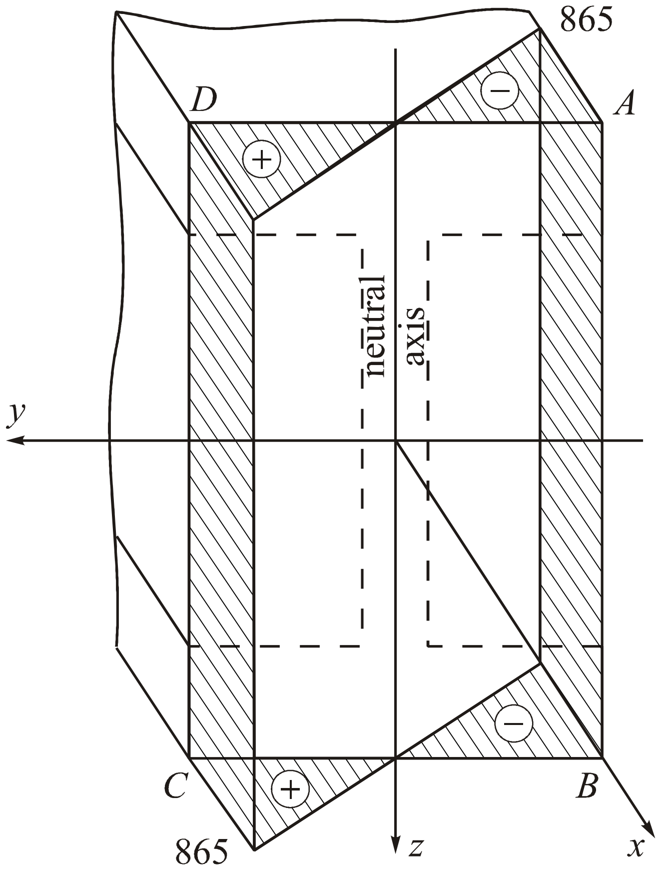


Fig. 7

General conclusion: *B* section is actually critical section of the beam in oblique bending and $|s_{max}| = 865\ \text{MPa}$**.**