MINISTRY OF EDUCATION AND SCIENCE OF UKRAINE

National aerospace university “Kharkiv Aviation Institute”

Department of aircraft strength

Course
Mechanics of materials and structures

HOME PROBLEM 12

Stress Analysis in Eccentric Tension – Compression

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Data of submission:

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National aerospace university
“Kharkiv Aviation Institute”
Department of aircraft strength

Subject: mechanics of materials
Document: home problem
Topic: stress analysis in eccentric tension – compression
Full name of the student, group

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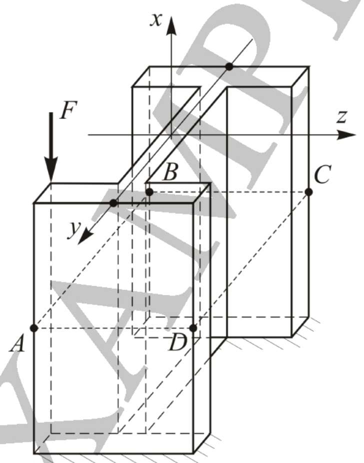

Given: *I*-beam №(18), F = 10 kN, $[\sigma] = 160$ MPa **.**

Goal: 1) calculate stresses in an arbitrary cross – section *ABCD* and check the strength; 2) Draw the graph of stress distribution in cross – section ABCD; 3) determine analytically position of neutral axis.

Full name of the lecturer **signature**

Mark:

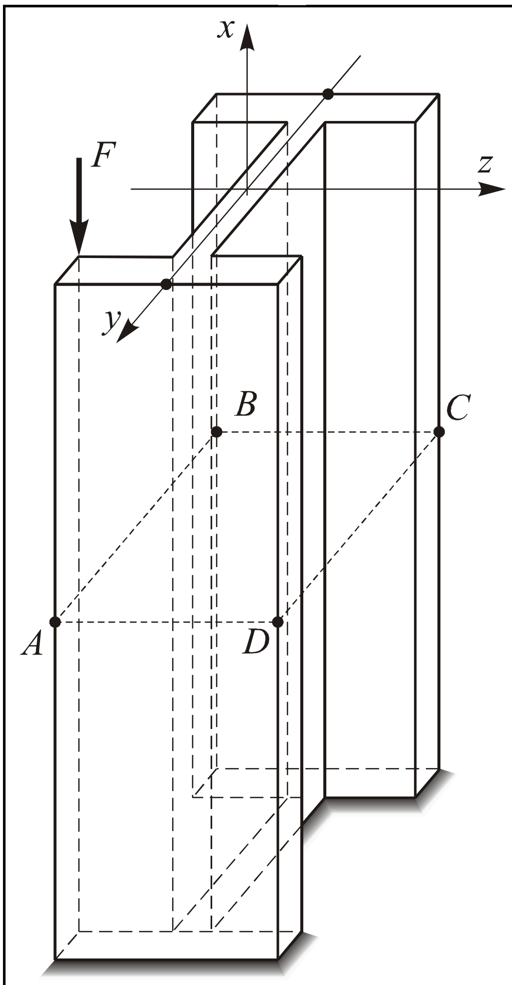

Fig. 1

Data:

1) Column is I-beam N18 with the following geometrical properties: $h = 18\times10^{-2}$ m, $b = 9\times10^{-2}$ m, $t = 8.1\times10^{-3}$ m (web), $d = 10^{-3}$ m (flange), $A = 23.4\times10^{-4}$ m^2, $I_y = 82.6\times10^{-8}$ m^4, $I_z = 1290\times10^{-8}$ m^4, $W_y = 18.4\times10^{-6}$ m^3, $W_z = 143\times10^{-6}$ m^3
2) External load $F = 10$ kN.
3) Allowable stresses $[\sigma]_t = 20$ MPa, $[\sigma]_c = 80$ MPa.

Goal:

1) Determine the internal forces in an arbitrary cross section *ABCD* far from the Saint-Venant zone.
2) Calculate acting stresses in potentially critical points of an arbitrary *ABCD* cross section.
3) Design the graph of stress distribution over the contour of *ABCD* section and determine the neutral axis position.
4) Determine the neutral axis position analytically.

Solution

1) Let us cut the column mentally in an arbitrary *ABCD* section according to the method of sections and determine internal forces in top and bottom cross sections of the column (see. Fig. 2):

$$|N_x| = F = 10 \text{ kN}, \quad |M_y| = F\frac{b}{2} = 450 \text{ Nm}, \quad |M_z| = F\left(\frac{h}{2} - d\right) = 819 \text{ Nm}.$$

2) Calculate the stresses in potentially critical angular points of *ABCD* section. Calculate preliminary the stress moduli generated by three internal forces:

$$|\sigma(N_x)| = \frac{10\times10^3}{23.4\times10^{-4}} = 4.27 \text{ MPa},$$

$$|\sigma_{\max}(M_y)| = \frac{M_y}{W_y} = \frac{450}{18.4\times10^{-6}} = 24.46 \text{ MPa},$$

$$|\sigma_{\max}(M_z)| = \frac{M_z}{W_z} = \frac{819}{143\times10^{-6}} = 5.73 \text{ MPa}.$$

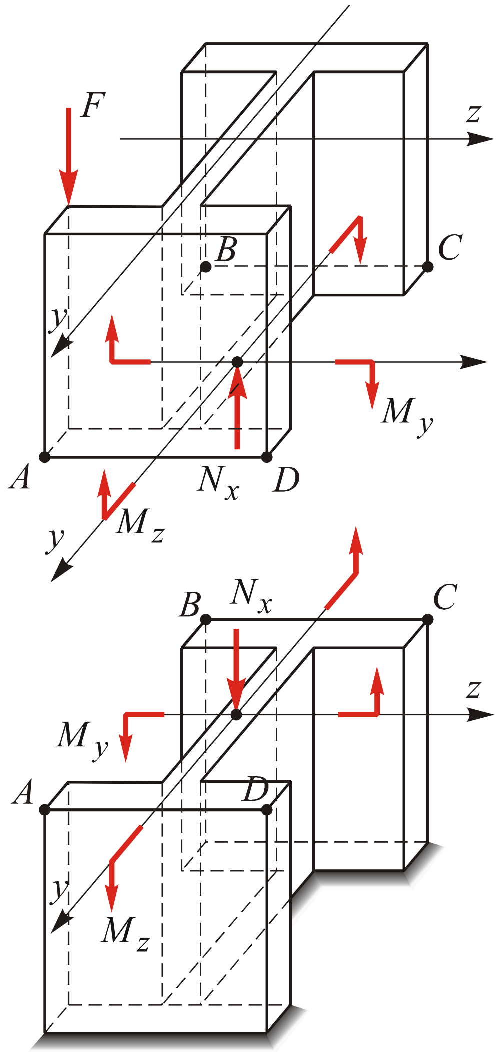

Fig. 2

Therefore,

$$\sigma_A = -\sigma(N_x) - \sigma_{\max}(M_y) - \sigma_{\max}(M_z) =$$
$$= -4.27 - 24.46 - 5.73 = -34.46 \text{ MPa},$$

$$\sigma_B = -\sigma(N_x) - \sigma_{\max}(M_y) + \sigma_{\max}(M_z) = -4.27 - 24.46 + 5.73 = -23.00 \text{ MPa},$$

$$\sigma_C = -\sigma(N_x) + \sigma_{\max}(M_y) + \sigma_{\max}(M_z) = -4.27 + 24.46 + 5.73 = 25.92 \text{ MPa},$$

$$\sigma_D = -\sigma(N_x) + \sigma_{\max}(M_y) - \sigma_{\max}(M_z) = -4.27 + 24.46 - 5.73 = 14.46 \text{ MPa}.$$

3) Design the graph of stress distribution in *ABCD* section in spatial cross section viewing with non-disturbed cross section for future geometrical analysis (see. Fig. 3).

4) Check the column strength:

(a) in tension, using the condition of strength

$\sigma_{\max_t} \le [\sigma]_t \to \sigma_{\max_t} = \sigma_C = 25.92 \text{ MPa} > 20 \text{ MPa}$.

Conclusion: the column in non-strong in tension.

(b) in compression, using the condition of strength

$\left|\sigma_{\max_c}\right| \le [\sigma]_c \to \left|\sigma_{\max_c}\right| = \left|\sigma_A\right| = 34.46 \text{ MPa} < 80 \text{ MPa}$.

Conclusion: the column is strong in compression.

General conclusion: the column is non-strong.

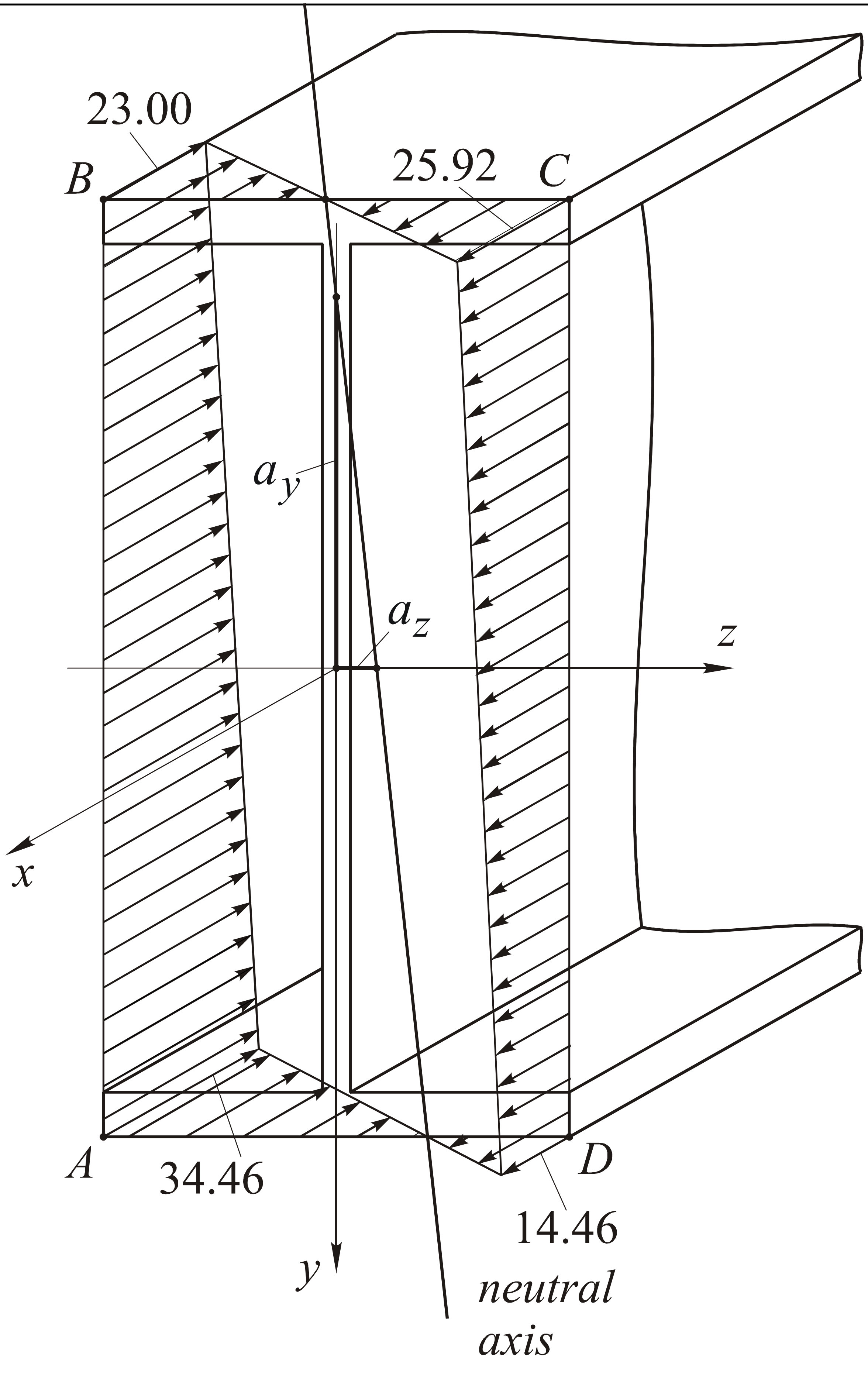

Рис. 3

5) Determine the neutral axis position analytically. For this purpose, equate to zero the stress in an arbitrary point *K* of *ABCD* section. Let us select this point in the first quarter of the section (see Fig. 4).

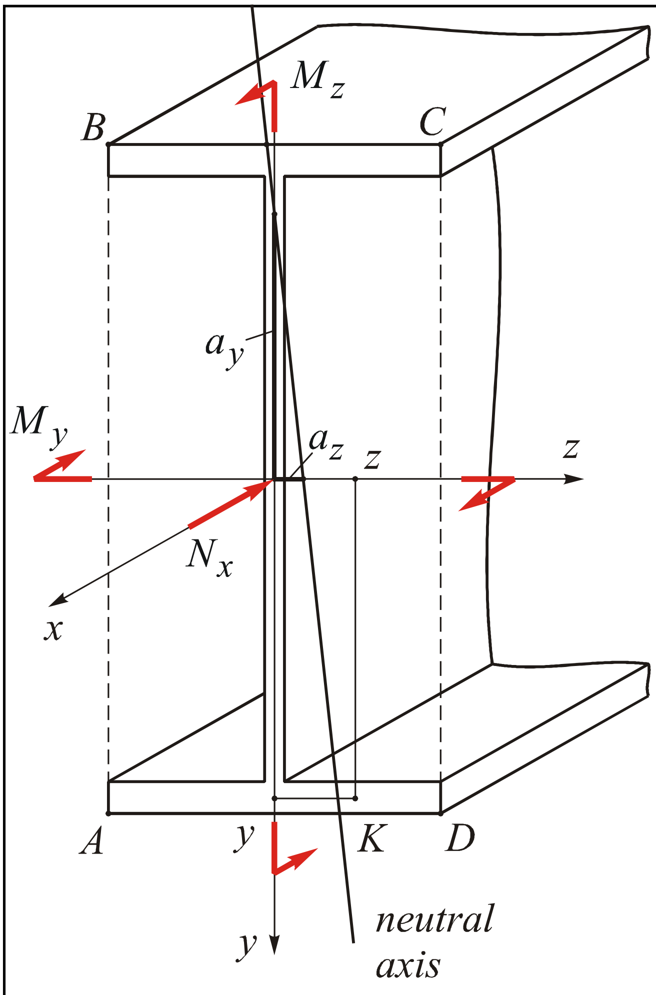

Fig. 4

$$\sigma_K = -\frac{N_x}{A} + \frac{M_y}{I_y} z - \frac{M_z}{I_z} y = 0$$

(this is equation of the plane in (σ, z, y) system of coordinates).

It may be rewritten as

$Az + By + C = 0$, where

$$A = \frac{M_y}{I_y} = \frac{450}{82.6\times10^{-8}} = 5.45\times10^{8},$$

$$B = -\frac{M_z}{I_z} = -\frac{819}{1290\times10^{-8}} = -0.63\times10^{8},$$

$$C = -\frac{N_x}{A} = -\frac{10\times10^{3}}{23.4\times10^{-4}} = -0.43\times10^{7}.$$

Find the segments which the neutral axis cuts on the coordinate axes:

(a) in $z = 0$

$$y^* = a_y = -\frac{C}{B} = -\frac{-0.43\times10^{7}}{-0.63\times10^{8}} = -0.683\times10^{-1}\text{m} =$$
$$= -68\times10^{-1}\ \text{m} = -68\ \text{mm};$$

(b) in $y = 0$

$$z^* = a_z = -\frac{C}{A} = -\frac{-0.43\times10^{7}}{5.45\times10^{8}} = +0.079\times10^{-1}\ \text{m} = +7.9\times10^{-3}\ \text{m} = +7.9\ \text{mm}.$$

These segments are shown in scaled cross section sketch on Fig. 4. Finally, the neutral axis is drawn through the segments tips.

6) Check the solution accuracy, correlating the neutral axis positions on Figs. 3 and 4.