MINISTRY OF EDUCATION AND SCIENCE OF UKRAINE

National aerospace university “Kharkiv Aviation Institute”

Department of aircraft strength

Course
Mechanics of materials and structures

HOME PROBLEM 13

Generalized Displacements in Cantilevers in Plane Bending

Name of student:

Group:

Advisor:

Data of submission:

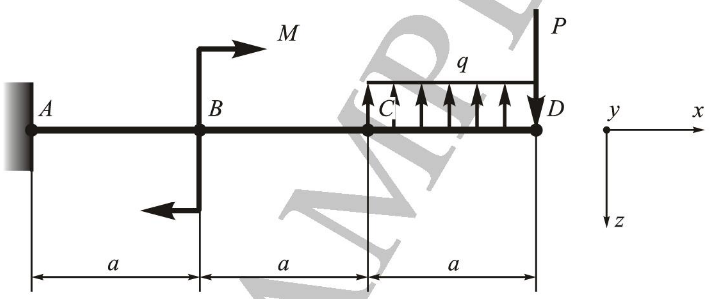
Mark:

National aerospace university
“Kharkiv Aviation Institute”
Department of aircraft strength

Subject: mechanics of materials
Document: home problem
Topic: Generalized Displacements in Cantilevers in Plane Bending.

Full name of the student, group

Variant: 4 **Complexity:** 1



Given: $q = 10$ kN/m; $P = 20$ kN; $M = 10$ kNm; $E = 2\times10^{11}$ Pa; $[\sigma] = 160$ MPa; $a = 2$ m.

Goal:
1) calculate dimensions of the cross-section choosing the one of following: a) diameter of the round solid; b) dimensions of the rectangle (*h/b*=2); c) I-beam number;
2) calculate vertical displacement and the slope in the following points:

$\theta_B - ?$	$\theta_C - ?$	$\theta_D - ?$
$z_B - ?$	$z_C - ?$	$z_D - ?$

signature

Full name of the lecturer

Mark:

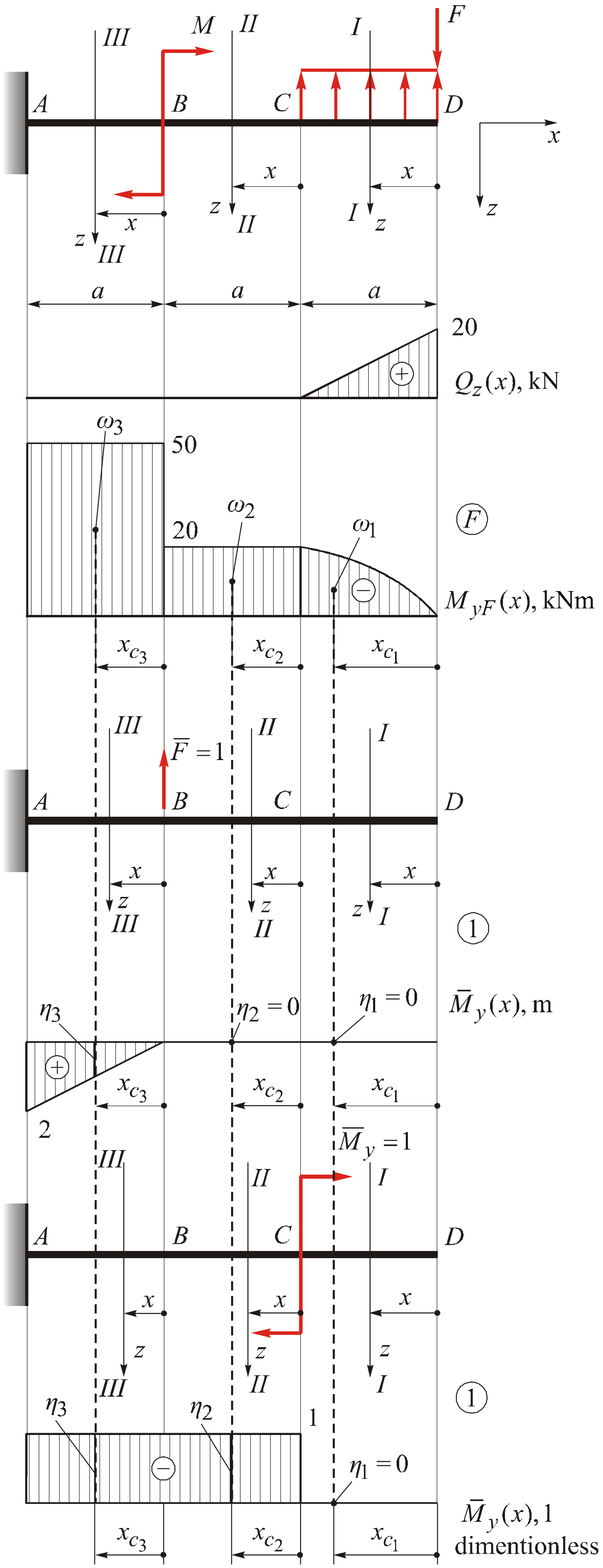


Fig. 1

Given:

$q = 10$ kN/m, $F = 20$ kN,

$M = 30$ kNm, $a = 2$ m,

$E = 2 \cdot 10^{11}$ Pa, $[\sigma] = 160$ MPa.

Required:

1) to determine the I-beam section dimensions;

2) to calculate the vertical deflection of B section and the angle of rotation of C section.

Perform the solution by Mohr's energy method and Vereshchagin's graphical method.

Note

In a plane bending, the Mohr's integral is:

$$\delta = \sum_{i=1}^{n} \int_{l_i} \frac{M_{yF}(x)\bar{M}_y(x)}{EI_y} dx,$$

where $M_{yF}(x)$ – bending moment in an arbitrary section of the force system (F), $\bar{M}_y(x)$ – bending moment in an arbitrary section of the unit system (1);

EI_y – flexural rigidity.

Vereshchagin's formula is:

$$\delta = \sum_{i=1}^{n} \frac{\omega_i \eta_i}{EI_y},$$ where ω_i – area of the bending moment graph part for the force system; η_i – ordinate of unit diagram under centroid of force system bending moment graph.

Solution

1. Writing the equations of internal forces in arbitrary sections of every part of given (force) system:

I – I $\quad 0 < x < a$

$$Q^{I}_{zF}(x) = F - qx\big|_{x=0} =$$

$$= 20\big|_{x=2} = 0 \text{ kN},$$

$$M_{yF}^{I}(x) = -Fx + \frac{qx^2}{2}\Bigg|_{x=0} = 0\Big|_{x=2} = -20 \text{ kNm}.$$

II – II $\quad 0 < x < a$

$$Q_{zF}^{II}(x) = F - qa = 0 \text{ kN},$$

$$M_{yF}^{II}(x) = -F(a + x) + qa\left(\frac{a}{2} + x\right)\Big|_{x=0} = -20\,\Big|_{x=2} = -20 \text{ kNm}.$$

III – III $\quad 0 < x < a$

$$Q_{zF}^{III}(x) = F - qa = 0 \text{ kN},$$

$$M_{yF}^{III}(x) = -F(2a + x) - M + qa\left(\frac{a}{2} + a + x\right)\Big|_{x=0} = -50\,\Big|_{x=2} = -50 \text{ kNm}.$$

2. Constructing diagrams of shear forces and bending moments of force system, and defining areas of the diagrams parts (see Fig. 1):

$$\omega_1 = \int_0^a M_{yF}^{I}(x)dx = \int_0^a \left(-20x + 5x^2\right)dx = -20\frac{x^2}{2}\Bigg|_0^2 + 5\frac{x^3}{3}\Bigg|_0^2 = -26{,}67 \text{ kNm}^2,$$

$$\omega_2 = \int_0^a M_{yF}^{II}(x)dx = (-20) \times 2 = -40 \text{ kNm}^2,$$

$$\omega_3 = \int_0^a M_{yF}^{III}(x)dx = (-50) \times 2 = -100 \text{ kNm}^2.$$

Knowing these areas is necessary for further determination of linear and angular displacements by Vereshchagin's method.

Conclusion: in III – III critical portion $\left|M_{y\max}\right| = 50$ kNm.

3. Determining the dimensions of I-beam cross section from condition of strength:

$$\sigma_{\max} = \frac{\left|M_{y\max}\right|}{W_y} \le [\sigma], \quad \rightarrow \quad W_y = \frac{\left|M_{y\max}\right|}{[\sigma]} = \frac{50 \times 10^3}{160 \times 10^6} = 312{,}5 \times 10^{-6} \text{ m}^3.$$

The nearest lower I-beam number N24 has $W_y = 289 \times 10^{-6}$ m^3.

Calculating the maximum acting stresses in I-beam N24:

$$\sigma_{\max} = \frac{50 \times 10^3}{289 \times 10^{-6}} = 173 \text{ MPa}.$$

Overstress $\Delta\sigma = \dfrac{\sigma_{\max} - [\sigma]}{[\sigma]} \times 100\% = \dfrac{173 - 160}{160} \times 100\% = 8{,}1\%$.

Since overstress is greater than 5%, I-beam N24 is unacceptable, therefore, we choose nearest biggest I-beam N24A: $W_y = 317 \times 10^{-6}$ m^3. For such I-beam $\sigma_{\max} = \dfrac{50 \times 10^3}{317 \times 10^{-6}} = 157.7$ MPa, i.e. this I-beam section is strong.

Let us copy from the assortment the axial moment of inertia of I-beam N24A:
$I_y = 3800\times10^{-8}$ m^4.

4. Determining the deflection at the B point, using Mohr's method. Given beam we will consider as force system (F), and will create corresponding unit system (1) by applying dimensionless unit force in B point in vertical direction, i.e. in the direction of deflection. Let us design the graph of bending moments for unit system (see Fig. 1). Simultaneously, let us simplify the equations of bending moments for (F) system in the most simple shape suitable for future integration. The equations of bending moments $M_{yF}(x)$ and $\bar{M}_y(x)$ are the following:

I – I $\quad 0 < x < 2$

$$M_{yF}^{I}(x) = -Fx + \frac{qx^2}{2} = -20x + 5x^2 \text{ kNm},$$

$$\bar{M}_y^{I}(x) = 0 \text{ kNm}.$$

II – II $\quad 0 < x < 2$

$$M_{yF}^{II}(x) = -F(a+x) + qa\left(\frac{a}{2}+x\right) = -20\times2 - 20x + 20 + 20x = -20 \text{ kNm},$$

$$\bar{M}_y^{II}(x) = 0.$$

III – III $\quad 0 < x < 2$

$$M_{yF}^{III}(x) = -F(2a+x) - M + qa\left(\frac{a}{2}+a+x\right) = -80 - 20x - 30 + 60 + 20x = -50 \text{ kNm},$$

$$\overline{M}_y^{III}(x) = 1x \text{ m}.$$

Substituting these pairs of equations into Mohr's integral and integrating, we obtain:

$$z_B = \frac{1}{EI_y}\left[\int_0^2 (-20x+5x^2)(0)dx + \int_0^2 (-20)(0)dx + \int_0^2 (-50)(x)dx\right] =$$

$$= -\frac{1}{EI_y}\left(50\frac{x^2}{2}\bigg|_0^2\right) = -\frac{100}{EI_y},\ \frac{\text{kNm}^3}{EI}.$$

Substituting the parameters of bending stiffness EI_y, we obtain

$$z_B = -\frac{100\times10^3}{2\times10^{11}\times3800\times10^{-8}} = -13.2\times10^{-3}\text{m} = -13.2 \text{ mm}.$$

Note, that sign (–) in solution means, that true direction of B point displacement is opposite to originally adopted, i.e. B point moves downwards.

To calculate the required displacement by Vereshchagin's method we should use bending moments graph $\overline{M}_y(x)$ to find its ordinates under the centroids of ω_1, ω_2 and ω_3 areas. It is designed on the Fig. 1, where these ordinates η_1, η_2, η_3 are shown. These ordinates are equal to $\eta_1 = 0$, $\eta_2 = 0$, $\eta_3 = +\frac{1}{2}\times2 = 1$ m.

Then

$$z_B = \frac{1}{EI}\left[\omega_1\eta_1 + \omega_2\eta_2 + \omega_3\eta_3\right] = \frac{1}{EI}\left[(-26{,}67)\times 0 + (-40)\times 0 + (-100)(+1)\right] = -\frac{100}{EI_y}\,,\ \frac{\text{kNm}^3}{EI_y}.$$

$z_B = -\dfrac{100\times 10^3}{2\times 10^{11}\times 3800\times 10^{-8}} = -13.2\times 10^{-3}$ m, which coincides with the result of calculation by Mohr's method.

5. Determining the angle of rotation of C section. Given beam also is considered as force system (F), and to design new unit system (1) we apply dimensionless unit moment $\overline{M} = 1$ in C section in arbitrary direction, clockwise, for example (see Fig. 1). Also, we draw corresponding diagram of unit bending moments $\overline{M}_y(x)$.

Dividing both systems onto portions in an identical manner, let us write equations of bending moments in the simplest shape, suitable for substituting into Mohr's integral:

I – I $\quad 0 < x < 2$

$$M_{yF}^{I}(x) = -Fx + \frac{qx^2}{2} = -20x + 5x^2 \text{ kNm},$$

$$\overline{M}_y^{I}(x) = 0 \text{ kNm}.$$

II – II $\quad 0 < x < 2$

$$M_{yF}^{II}(x) = -F(a + x) + qa\left(\frac{a}{2} + x\right) = -20 \text{ kNm},$$

$\overline{M}_y^{II}(x) = -1$, dimensionless.

III – III $\quad 0 < x < 2$

$$M_{yF}^{III}(x) = -F(2a + x) - M + qa\left(\frac{a}{2} + a + x\right) = -50 \text{ kNm},$$

$\overline{M}_y^{III}(x) = -1$, dimensionless.

Substituting these equations into Mohr's integral in pairs and integrating, we obtain:

$$\theta_C = \frac{1}{EI}\left[\int_0^2 (-20x + 5x^2)(0)dx + \int_0^2 (-20)(-1)dx + \int_0^2 (-50)(-1)dx\right] = +\frac{140}{EI_y},\ \frac{\text{kNm}^3}{EI_y}.$$

Substituting the parameters of flexural rigidity EI_y, we obtain

$$\theta_C = +\frac{140\times 10^3}{2\times 10^{11}\times 3800\times 10^{-8}} = 18.4\times 10^{-3}, \text{rad} = 1.05^\circ.$$

Note, that (+) sign in solution means that *C* section is really rotates clockwise.

To calculate the required displacement by Vereshchagin's method let us use the existing bending moment diagram of force system to take areas ω_1, ω_2, ω_3. Under these areas centroids we calculate new ordinates η_1, η_2, η_3 on the diagram of new unit system (see Fig. 1). In our case, $\eta_1 = 0$, $\eta_2 = 1$, $\eta_3 = -1$ – dimensionless value. Then

$$\theta_C = \frac{1}{EI}\left[\omega_1\eta_1 + \omega_2\eta_2 + \omega_3\eta_3\right] = \frac{1}{EI_y}\left[(-26,67)\times 0 + (-40)\times(-1) + (-100)\times(-1)\right] =$$

$$= +\frac{140}{EI_y},\ \frac{\text{kNm}^3}{EI_y}.$$

Finally, after substituting the flexural rigidity parameters we find

$$\theta_C = +\frac{140\times 10^3}{2\times 10^{11}\times 3800\times 10^{-8}} = 18.4\times 10^{-3},\ \text{rad} = 1.05^\circ.$$

This result coincides with calculation by Mohr’s method.