MINISTRY OF EDUCATION AND SCIENCE OF UKRAINE

National aerospace university “Kharkiv Aviation Institute”
Department of aircraft strength

Course
Mechanics of materials and structures

HOME PROBLEM 18

Buckling and Stability of Compressed Rods

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National aerospace university
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Department of aircraft strength

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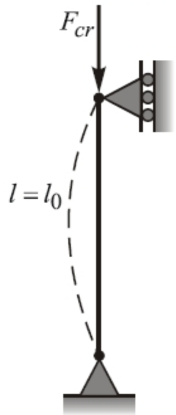
Topic: Buckling and Stability of Compressed Rods.
Full name of the student, group

Variant: 1 **Complexity:** 1



$l_0 = l$
$\nu = 1$

Given: $l = 3$ m, cross-section: I-beam, $F = 100$ kN.

Goal: 1) determine the cross-sectional dimensions; 2) calculate the value of critical force for selected column; 3) calculate the value of allowable load for selected column.

signature

Full name of the lecturer

Mark:

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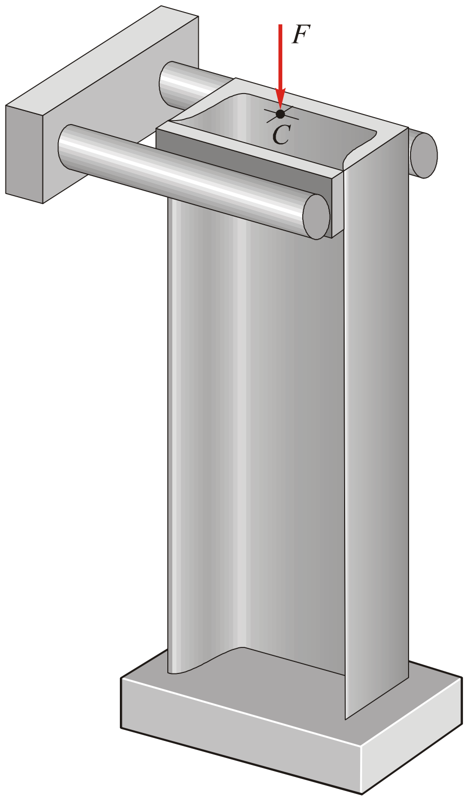

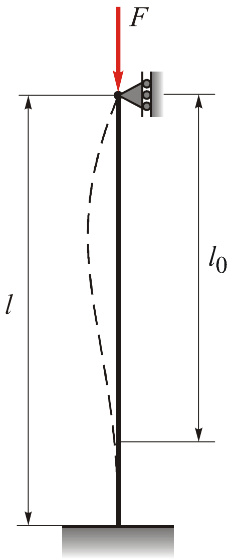

Fig. 1

Data: cross-section type – channel (see Fig. 1), $l = 2.5\,\text{m}$, $s_{pr} = 250\,\text{MPa}$, $[s]_c = 160\,\text{MPa}$, $F = 150\,\text{kN}$.

Goal:
1) Determine channel section number from the condition of stability.
2) For selected number, calculate critical force value F_{cr}.
3) Determine actual value of the safety factor n_y
4) For selected number, calculate allowable force value $[F]$.

Solution

1. Selecting the channel section number from the condition of stability.
Condition of stability is the following:

$$s = \frac{F}{A} \le [s]_s, \text{ where}$$

$[s]_s$ – allowable stress for stability:

$$[s]_s = j\,[s]_c, \text{ where}$$

j – stress reduction factor, A – unknown cross-sectional area, F – compressive force.

Since j factor is the tabular function of the post slenderness ratio l, and the last value is determined by unknown cross-sectional dimensions, this problem will be solved by approximation method using available j range: $0 < j < 1$. In the first approach, we will assume that $j^I = 0.5$.

I-st approach

Calculating the area from the condition of stability:

$$A^I \ge \frac{F}{j^I [s]_c} = \frac{150\times10^3}{0.5\times160\times10^6} = 18.75\times10^{-4}\,\text{m}^2 = 18.75\,\text{cm}^2.$$

Applying the channel section assortment, let us determine the closest number of channel section and find it's minimal geometrical properties, i.e. the moment and radius of inertia in the plain of maximum slenderness:

channel No 16: $A = 18.1\,\text{cm}^2$, $I_{\min} = 63.3\,\text{cm}^4$, $i_{\min} = 1.87\,\text{cm}$.

Knowing the length reduction factor of the post $n = 0.7$, let us calculate it's maximum slenderness ratio:

$$l_{\max}^{I} = \frac{n l}{i_{\min}} = \frac{0.7 \times 2.5}{1.87 \times 10^{-2}} = 93.6 .$$

Using this value in the table of stress reduction factor dependence on slenderness ratio, we will determine actual stress reduction factor value applying linear interpolation:

$$j\big|_{l=90} = 0.69, \quad j\big|_{l=100} = 0.60, \quad j\big|_{l=93.6} = 0.69 - \frac{0.69 - 0.60}{10} \times 3.6 = 0.6576 .$$

After calculation, new value of the stress reduction factor becomes equal

$$j^{I*} = 0.6576 .$$

Comparing $j^{I} = 0.5$ and $j^{I*} = 0.6576$, we conclude that the error is:

$$D = \frac{j^{I*} - j^{I}}{j^{I}} \times 100\% = 31.52\% > 5\% .$$

Since the error exceeds 5% range ($D > 5\%$), it will be necessary to perform second approach assuming

$$j^{II} = \frac{j^{I} + j^{I*}}{2} = \frac{0.5 + 0.6576}{2} = 0.5788 .$$

II-nd approach

Cross-sectional area in this approach is

$$A^{II} = \frac{F}{j^{II}[s]_c} = \frac{150 \times 10^{3}}{0.5788 \times 160 \times 10^{6}} = 16.2\,\text{cm}^2 .$$

The closest channel number is No 14: $A^{II} = 15.6\,\text{cm}^2$, $I_{\min} = 45.4\,\text{cm}^4$, $i_{\min} = 1.7\,\text{cm}$.

It's maximum slenderness ratio is:

$$l_{\max}^{II} = \frac{n l}{i_{\min}} = \frac{0.7 \times 2.5}{1.7 \times 10^{-2}} = 102.9 .$$

Applying the table of stress reduction factor dependence on slenderness ratio, we will determine actual stress reduction factor value also applying linear interpolation:

$$j\big|_{l=100} = 0.60, \quad j\big|_{l=110} = 0.52,$$

$$j\big|_{l=102.9} = 0.60 - \frac{0.60 - 0.52}{10} \times 2.9 = 0.5768 = j^{II*} .$$

Comparing new (j^{II*}) and old (j^{II}) values shows that

$$D = \frac{j^{II*} - j^{II}}{j^{II}} \times 100\% = \frac{0.5768 - 0.5788}{0.5788} \times 100\% = 0.35\% < 5\% .$$

Due to $D < 5\%$, our solution is successful. Selected channel No 14 has the following geometrical properties:

$$A = 15.6\,\text{cm}^2,\ I_{\min} = 45.4\,\text{cm}^4,\ i_{\min} = 1.7\,\text{cm},\ l_{\max} = 102.9,\ j = 0.5768 .$$

2. Calculating the critical force value of the No 14 channel post.

First of all, let us determine limiting slenderness ratio of the post material (carbon steel):

$$l_{\lim} = p\sqrt{\frac{E}{s_{pr}}} = 3.14\sqrt{\frac{2\times10^{11}}{250\times10^{6}}} = 88.8.$$

Since $l_{\max} > l_{\lim}$ $(102.9 > 88.8)$, Euler's formula will be used for critical force calculating:

$$F_{cr} = \frac{p^2 EI_{\min}}{(n\,l)^2} = \frac{3.14^2\times2\times10^{11}\times45.4\times10^{-8}}{(0.7\times2.5)^2} = 292.6\,\text{kN}.$$

3. Calculating actual factor of safety for selected post No 14.

$$n_{st} = \frac{F_{cr}}{F} = \frac{292.6}{150} = 1.95.$$

4. Calculating the allowable value of external force for No 14 channel post.
For this purpose, we will use the condition of stability

$$s = \frac{F}{A} \le j\,[s]_c.$$

Therefore, $[F] = Aj\,[s]_c = 15.6\times10^{-4}\times0.5768\times160\times10^{6} = 144\,\text{kN}.$

Conclusion

Calculated actual factor of safety for selected post $n_{st} = 1.95$ exceeds the value of safety factor in compression ($n = 1.5 - 1.7$) due to high danger of buckling failure. This fact is the result of corresponding j factor selection in regulation documents of civil engineering.